3 Inferring Stellar Luminosity

3.1 "Standard Candle" methods for distance

In our everyday experience, there is another way we sometimes infer distance, namely by the change in apparent brightness for objects that emit their own light, with some known power or "luminosity". For example, a hundred watt light bulb at a distance of $d = 1$ m certainly appears a lot brighter than that same bulb at $d = 100$ m. Just as for a star, what we observe as apparent brightness is really a measure of the *flux* of light, i.e. energy per unit time *per unit area* (erg/s/cm^2 in CGS units, or watt/m^2 in MKS).

When viewing a light bulb with our eyes, it's just the rate at which the light's energy is captured by the area of our pupils. If we assume the light bulb's emission is *isotropic* (i.e., the same in all directions), then as the light travels outward to a distance d, its power or luminosity is spread over a sphere of area $4\pi d^2$. This means that the light detected over a fixed detector area (like the pupil of our eye, or, for telescopes observing stars, the area of the telescope mirror) decreases in proportion to the *inverse-square* of the distance, $1/d^2$. We can thus define the apparent brightness in terms of the flux,

$$\boxed{F = \frac{L}{4\pi d^2}} \; . \tag{3.1}$$

This is a profoundly important equation in astronomy, and so you should not just memorize it, but embed it completely and deeply into your psyche.

In particular, it should become obvious that this equation can be readily used to infer the distance to an object of *known luminosity*, an approach called the *standard candle* method. (Taken from the idea that a candle, or at least a "standard" candle, has a known luminosity or intrinisic brightness.) As discussed further in sections below, there are circumstances in which we can get clues to a star's (or other object's) intrinsic luminosity L, for example through careful study of a star's spectrum. If we then measure the apparent brightness (i.e. flux F), we can infer the distance through:

$$d = \sqrt{\frac{L}{4\pi F}} \; . \tag{3.2}$$

Indeed, when the study of a stellar spectrum is the way we infer the luminosity, this method of distance determination is sometimes called "spectroscopic parallax".

Of course, if we can independently determine the distance through the actual trigonometric parallax, then such a simple measurement of the flux can instead be used to determine the luminosity,

$$L = 4\pi d^2 \, F \, . \tag{3.3}$$

In the case of the Sun, the flux measured at Earth is referred to as the "solar constant", with a measured mean value of about

$$F_\odot \approx 1.4 \, \frac{\text{kW}}{\text{m}^2} = 1.4 \times 10^6 \, \frac{\text{erg}}{\text{cm}^2 \, \text{s}} \, . \tag{3.4}$$

If we then apply the known mean distance of the Earth to the Sun, $d = 1$ au, we obtain for the solar luminosity

$$L_\odot \approx 4 \times 10^{26} \text{W} = 4 \times 10^{33} \, \frac{\text{erg}}{\text{s}} \, . \tag{3.5}$$

Thus we see that the Sun emits the power of about 4×10^{24} 100-watt light bulbs! In common language this corresponds to four million billion billion, a number so huge that it loses any meaning. It illustrates again how in astronomy we have to think on a entirely different scale than we are used to in our everyday world.

But once we get used to the idea that the luminosity and other properties of the Sun are huge but still finite and measurable, we can use these as benchmarks for characterizing analogous properties of other stars and astronomical objects. In the case of stellar luminosities, for example, these typically range from about $L_\odot/1000$ for very cool, low-mass "dwarf" stars, to as high as $10^6 L_\odot$ for very hot, high-mass "supergiants".

As discussed further below, the luminosity of a star depends directly on both its size (i.e. radius) and surface temperature. But more fundamentally these in turn are largely set by the star's mass, age, and chemical composition.

3.2 Intensity or Surface Brightness

For any object with a resolved solid angle Ω, an important flux-related quantity is the *surface brightness* – also known as the specific intensity I; this can be roughly (though not quite exactly; see §12.1) thought of as the *flux per solid angle*, i.e.

$$I \approx \frac{F}{\Omega} \approx \frac{L}{4\pi d^2 \pi (R/d)^2} \approx \frac{L}{4\pi^2 R^2} = \frac{F_*}{\pi} \, , \tag{3.6}$$

where $F_* \equiv F(R) = L/4\pi R^2$ is the *surface flux* evaluated at the stellar radius R. As illustrated in figure 3.1, the surface brightness of any resolved radiating object turns out, somewhat surprisingly, to be *independent of distance*. This is because, even though the flux declines with distance, the surface brightness 'crowds' this

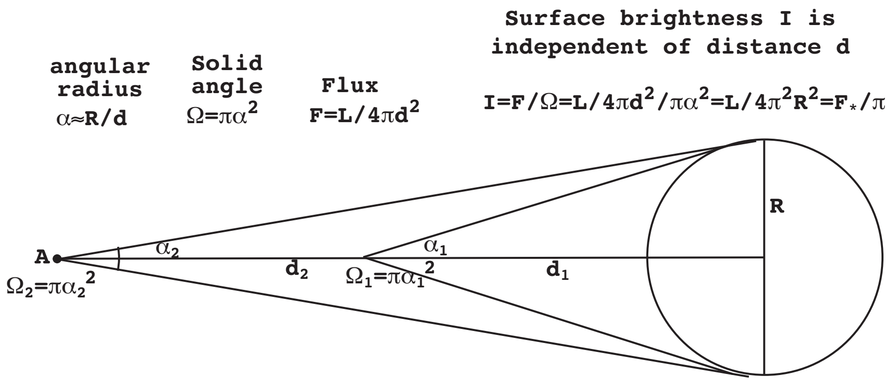

Figure 3.1 Distance independence of surface brightness of a radiating sphere, representing the flux per solid angle, $B = F/\Omega$. At greater distance d, the flux declines in proportion to $1/d^2$; but because this flux is squeezed into a smaller solid angle Ω, which also declines as $1/d^2$, the surface brightness B remains constant, independent of the distance.

flux into a proportionally smaller solid angle as the distance is increased. The ratio of flux per solid angle, or surface brightness, is thus constant.

In particular, if we ignore any absorption from earth's atmosphere, the surface brightness of the Sun that we see here on earth is actually the *same* as if we were standing on the surface of the Sun itself!

Of course, on the surface of the Sun, its radiation will fill up half the sky – i.e. 2π steradians, instead of the mere $0.2\,\text{deg}^2 = 6 \times 10^{-5}$ steradians seen from earth. The huge flux from this large, bright solid angle would cause a lot more than a mere sunburn![1]

3.3 Apparent and absolute magnitude and the distance modulus

To summarize, we have now identified 3 distinct kinds of "brightness" – absolute, apparent, and surface – associated respectively with the luminosity (energy/time), flux (energy/time/area), and specific intensity (flux emitted into a given solid angle). Before moving on to examine additional properties of stellar radiation, let us first discuss some specifics of how astronomers characterize apparent vs. absolute brightness, namely through the so-called "magnitude" system.

This system has some rather awkward conventions, developed through its long history, dating back to the ancient Greeks. As noted in §1, they ranked the apparent brightness of stars in 6 bins of magnitude, ranging from $m = 1$ for the brightest to $m = 6$ for the dimmest. Because the human eye is adapted to

[1] NASA 's recently launched "Parker Solar Probe" will eventually fly within about $9R_\odot$ of the solar surface, or about $\sim$1/20 au. So a key challenge has been to provide the shielding to keep the factor >400 higher solar radiation flux from frying the spacecraft's instruments.

detect a large dynamic range in brightness, it turns out that our perception of brightness depends roughly on the *logarithm* of the flux.

In our modern calibration this can be related to the Greek magnitude system by stating that a *difference of 5* in magnitude represents a *factor 100* in the relative brightness of the compared stars, with the *dimmer* star having the *larger magnitude*. This can be expressed in mathematical form as

$$\boxed{m_2 - m_1 = 2.5\ \log(F_1/F_2)\,.} \tag{3.7}$$

We can further extend this logarithmic magnitude system to characterize the absolute brightness, a.k.a. luminosity, of a star in terms of an *absolute* magnitude. To remove the inherent dependence on distance in the flux F, and thus in the apparent magnitude m, the absolute magnitude M is defined as the apparent magnitude that a star *would* have if it were placed at a standard distance, chosen by convention to be $d = 10\,\text{pc}$. Since the flux scales with the inverse-square of distance, $F \sim 1/d^2$, the difference between apparent magnitude m and absolute magnitude M is given by

$$\boxed{m - M = 5\log(d/10\,\text{pc})}\,, \tag{3.8}$$

which is known as the *distance modulus*.

The absolute magnitude of the Sun is $M \approx +4.8$ (though for simplicity in calculations, this is often rounded up to 5), and so the scaling for other stars can be written as

$$\boxed{M = 4.8 - 2.5\ \log(L/L_\odot)}\,. \tag{3.9}$$

Combining these relations, we see that the apparent magnitude of any star is given in terms of the luminosity and distance by

$$\boxed{m = 4.8 - 2.5\ \log(L/L_\odot) + 5\ \log(d/10\,\text{pc})\,.} \tag{3.10}$$

For bright stars, magnitudes can even become negative. For example, the (apparently) brightest star in the night sky, Sirius, has an apparent magnitude $m = -1.42$. But with a luminosity of just $L \approx 23L_\odot$, its absolute magnitude is still positive, $M = +1.40$. Its distance modulus, $m - M = -1.42 - 1.40 = -2.82$, is negative. Through eqn. (3.8), this implies that its distance, $d = 10^{1-2.82/5} = 2.7$ pc, is *less* than the standard distance of 10 pc used to define absolute magnitude and distance modulus [eqn. (3.8)].

3.4 Questions and Exercises

Quick Question 1: Recalling the relationship between an AU and a parsec from eqn. (2.6), use eqns. (3.8) and (3.9) to compute the apparent magnitude of the Sun. What then is the Sun's distance modulus?

Quick Question 2: Suppose two stars have a luminosity ratio $L_2/L_1 = 100$.