5 Inferring Stellar Radius from Luminosity and Temperature

We see from figure 4.2 that, in addition to a shift toward shorter peak wavelength λ_{max}, a higher temperature also increases the overall brightness of blackbody emission at *all* wavelengths. This suggests that the total energy emitted over all wavelengths should increase quite sharply with temperature. Leaving the details as an exercise for the reader, let us quantify this expectation by carrying out the necessary spectral integrals to obtain the temperature dependence of the *Bolometric* intensity of a blackbody

$$B(T) \equiv \int_0^\infty B_\lambda(T)\, d\lambda = \int_0^\infty B_\nu(T)\, d\nu = \frac{\sigma_{sb} T^4}{\pi}\,, \tag{5.1}$$

with $\sigma_{sb} = 2\pi^5 k^4/(15h^3c^2)$ known as the Stefan-Boltzmann constant, with numerical value $\sigma_{sb} = 5.67 \times 10^{-5}\,\mathrm{erg/cm^2/s/K^4} = 5.67 \times 10^{-8}\,\mathrm{J/m^2/s/K^4}$.

If we spatially resolve a pure blackbody with surface temperature T, then $B(T)$ represents the Bolometric *surface brightness* we would observe from each part of the visible surface.

5.1 Stefan-Boltzmann law for surface flux from a blackbody

Combining eqns. (3.6) and (5.1), we see that the radiative *flux* at the surface radius R of a blackbody is given by

$$\boxed{F_* \equiv F(R) = \pi\, B(T) = \sigma_{sb} T^4}\,, \tag{5.2}$$

which is known as the *Stefan-Boltzman law*.

The Stefan-Boltzmann law is one of the linchpins of stellar astronomy. If we now relate the surface flux to the stellar luminosity L over the surface area $4\pi R^2$, then applying this to the Stefan-Boltzmann law gives

$$\boxed{L = \sigma_{sb} T^4\, 4\pi R^2\,,} \tag{5.3}$$

which is often more convenient to scale by associated solar values,

$$\frac{L}{L_\odot} = \left(\frac{T}{T_\odot}\right)^4 \left(\frac{R}{R_\odot}\right)^2 . \tag{5.4}$$

We can also use eqn. (5.3) to solve for the stellar radius,

$$R = \sqrt{\frac{L}{4\pi\sigma_{sb}T^4}} = \sqrt{\frac{F(d)}{\sigma_{sb}T^4}}\, d\,, \tag{5.5}$$

where the latter equation uses the inverse-square-law to relate the stellar radius to the flux $F(d)$ and distance d, along with the surface temperature T.

For a star with a known distance d, e.g. by a measured parallax, measurement of apparent magnitude gives the flux $F(d)$, while measurement of the peak wavelength λ_{max} or color (e.g. B-V) provides an estimate of the temperature T (see figure 4.3). Applying these in eqn. (5.5), we can thus obtain an estimate of the stellar radius R.

5.2 Questions and Exercises

Quick Question 1: Compute the luminosity L (in units of the solar luminosity $L_\odot$), absolute magnitude M, and peak wavelength λ_{max} (in nm) for stars with (a) $T = T_\odot$; $R = 10R_\odot$, (b) $T = 10T_\odot$; $R = R_\odot$, and (c) $T = 10T_\odot$; $R = 10R_\odot$. If these stars all have a parallax of $p = 0.001$ arcsec, compute their associated apparent magnitudes m.

Quick Question 2: Suppose a star has a parallax $p = 0.01$ arcsec, peak wavelength $\lambda_{max} = 250$ nm, and apparent magnitude $m = +5$. About what is its:

a. Distance d (in pc)?
b. Distance modulus $m - M$?
c. Absolute magntidue M?
d. Luminosity L (in $L_\odot$)?
e. Surface temperature T (in $T_\odot$)?
f. Radius R (in $R_\odot$)?
g. Angular radius α (in radian and arcsec)?
h. Solid angle Ω (in steradian and $arcsec^2$)?
i. Surface brightness relative to that of the Sun, $B/B_\odot$?