8 Stellar Ages and Lifetimes

In our list of basic stellar properties, let us next consider stellar age. Just how old are stars like the Sun? What provides the energy that keeps them shining? And what will happen to them as they exhaust various available energy sources?

8.1 Shortness of chemical burning timescale for Sun and stars

When 19th century scientists pondered the possible energy sources for the Sun, some first considered whether this could come from the kind of chemical reactions (e.g., from fossil fuels like coal, oil, natural gas, etc.) that power human activities on Earth. But such chemical reactions involve transitions of electrons among various bound states of atoms, and, as discussed below (§A.1) for the Bohr model of the Hydrogen, the scale of energy release in such transitions is limited to something on the order of an electron volt (eV). In contrast, the rest-mass energy of the protons and neutrons that make up the mass is about 1 GeV, or 10^9 times higher. With the associated mass-energy efficiency of $\epsilon \sim 10^{-9}$, we can readily estimate a timescale for maintaining the solar luminosity from chemical reactions,

$$t_{chem} = \epsilon \, \frac{M_\odot c^2}{L_\odot} = \epsilon \; 4.5 \times 10^{20}\,\mathrm{s} = \epsilon \, 1.5 \times 10^{13}\,\mathrm{yr} \approx 15,000\,\mathrm{yr}\,. \tag{8.1}$$

Even in the 19th century, it was clear, e.g. from geological processes like erosion, that the Earth – and so presumably also the Sun – had to be much older than this.

8.2 Kelvin-Helmholtz timescale for gravitational contraction

So let us consider whether, instead of chemical reactions, gravitational contraction might provide the energy source to power the Sun and other stars. As a star undergoes a contraction in radius, its gravitational binding becomes stronger, with a deeper gravitational potential energy, yielding an energy release set by the negative of the change in gravitational potential ($-dU > 0$). If the contraction is gradual enough that the star roughly maintains dynamical equilibrium, then just half of the gravitational energy released goes into heating up the star[1], leaving the other half available to power the radiative luminosity, $L = -\frac{1}{2}dU/dt$. For a star of observed luminosity L and present-day gravitational binding energy U, we can thus define a characteristic gravitational contraction lifetime,

$$t_{grav} = -\frac{1}{2}\frac{U}{L} \equiv t_{KH} \tag{8.2}$$

where the subscript "KH" refers to Kelvin and Helmholtz, the names of the two scientists credited with first identifying this as an important timescale. To estimate a value for the gravitational binding energy, let us consider the example for the Sun under the somewhat artificial assumption that it has a uniform, constant density, given by its mass over volume, $\rho = M_\odot/(4\pi R_\odot^3/3)$. Since the gravity at any radius r depends only on the mass $m = \rho 4\pi r^3/3$ *inside* that radius, the total gravitational binding energy of the Sun is given by integrating the associated local gravitational potential $-Gm/r$ over all differential mass shells dm,

$$-U = \int_0^{M_\odot} \frac{Gm}{r}\, dm = \frac{16\pi^2}{3} G\rho^2 \int_0^R r^4\, dr = \frac{3}{5}\frac{GM_\odot^2}{R_\odot}\,, \tag{8.3}$$

Applying this in eqn. (8.2), we find for the "Kelvin-Helmholtz" time of the Sun,

$$t_{KH} \approx \frac{3}{10}\frac{GM_\odot^2}{R_\odot L_\odot} \approx 30\,\mathrm{Myr}\,. \tag{8.4}$$

Although substantially longer than the chemical burning timescale (8.1), this is still much shorter than the geologically inferred minimum age of the Earth, which is several *Billion* years.

8.3 Nuclear burning timescale

We now realize, of course, that the ages and lifetimes of stars like the Sun are set by a much longer *nuclear burning* timescale. When four hydrogen nuclei are fused into a helium nucleus, the helium mass is about 0.7% *lower* than the original four hydrogen. For nuclear fusion the above-defined mass-energy burning efficiency is thus now $\epsilon_{nuc} \approx 0.007$. But in a typical main sequence star, only some core fraction $f \approx 1/10$ of the stellar mass is hot enough to allow Hydrogen fusion. Applying this we thus find for the nuclear burning timescale

$$\boxed{t_{nuc} = \epsilon_{nuc}\, f\, \frac{Mc^2}{L} \approx 10 \text{ Gyr}\, \frac{M/M_\odot}{L/L_\odot}\,,} \tag{8.5}$$

where Gyr $\equiv 10^9$ yr, i.e., a billion years, or a "Giga-year".

[1] This is another example of the Virial theorem for gravitationally bound systems, as discussed in 7.4.

We thus see that the Sun can live for about 10 Gyr by burning Hydrogen into Helium in its core. It's present age of 4.6 Gyr[2] thus puts it roughly half way through this Hydrogen-burning phase, with about 5.4 Gyr to go before it runs out of H in its core.

8.4 Age of stellar clusters from main-sequence turnoff point

As discussed below (see §10.4 and eqn. 10.11), observations of stellar binary systems indicate that the luminosities of main-sequence stars scale with a high power of the stellar mass – roughly $L \sim M^3$. In the present context, this implies that high-mass stars should have much shorter lifetimes than low-mass stars.

If we make the reasonable assumption that the same fixed fraction ($f \approx 0.1$) of the total hydrogen mass of any star is available for nuclear burning into helium in its stellar core, then the fuel available scales with the mass, while the burning rate depends on the luminosity. Normalized to the Sun, the main-sequence lifetime thus scales as

$$\boxed{t_{ms} = t_{ms,\odot} \frac{M/M_\odot}{L/L_\odot} \approx 10\,\mathrm{Gyr} \left(\frac{M_\odot}{M}\right)^2 .} \tag{8.6}$$

The most massive stars, of order $100\,M_\odot$, and thus with luminosities of order $10^6 L_\odot$, have main-sequence lifetimes of only about about 1 Myr, much shorter the multi-Gyr timescale for solar-mass stars.

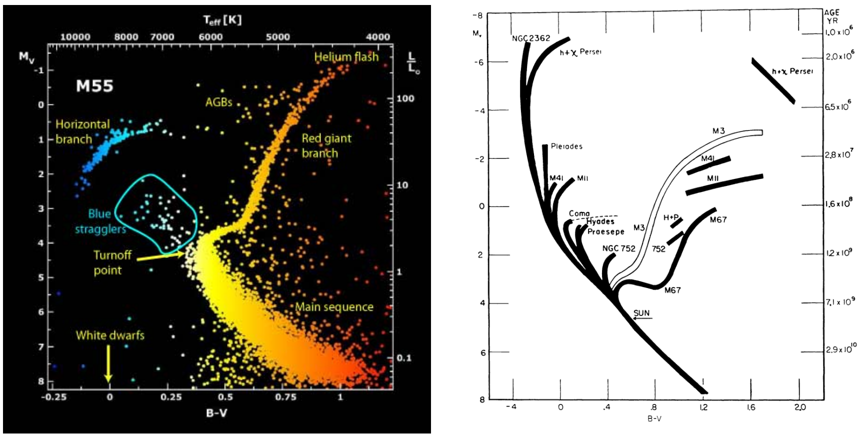

Figure 8.1 Left: H-R diagram for globular cluster M55, showing how stars on the upper main sequence have evolved to lower temperature giant stars. Right: Schematic H-R diagram for clusters, showing the systematic peeling off of the main sequence with increasing cluster age.

[2] As inferred, e.g., from radioactive dating of the oldest meteorites.

This strong scaling of lifetime with mass can be vividly illustratied by plotting the H-R diagram of stellar clusters. The H-R diagram plotted in figure 6.5 is for volume-limited sample near the Sun, consisting of stars of a wide range of ages, distances, and perhaps even chemical composition. But stars often appear in clusters, all roughly at the same distance, and, since they likely formed over a relatively short time span out of the same interstellar cloud, they all have roughly the same age and chemical composition. Using eqn. (8.6) together with the the $L \sim M^3$ relation, the age of a stellar cluster can be inferred from its H-R diagram simply by measuring the luminosity L_{to} of stars at the "*turn-off*" point from the main sequence,

$$\boxed{t_{cluster} \approx 10\,\mathrm{Gyr}\left(\frac{L_\odot}{L_{to}}\right)^{2/3}} \, . \qquad (8.7)$$

The left panel of figure 8.1 plots an actual H-R diagram for the globular cluster M55. Note that stars to the upper left of the main sequence have evolved to a vertical branch of cooler stars extending up to the Red Giants[3]. This reflects the fact that more luminous stars exhaust their hydrogen fuel sooner that dimmer stars, as shown by the inverse luminosity scaling of the nuclear burning timescale in eqn. (8.5). The right panel illustrates schematically the H-R diagrams for various types of stellar clusters, showing how the turnoff point from the main sequence is an indicator of the cluster age. Observed cluster H-R diagrams like this thus provide a direct diagnostic of the formation and evolution of stars with various masses and luminosities.

8.5 Questions and Exercises

Quick Question 1: What are the luminosities (in $L_\odot$) and the expected main sequence lifetimes (in Myr) of stars with masses: a. $10\,M_\odot$? b. $0.1\,M_\odot$? c. $100\,M_\odot$?

Quick Question 2: Suppose you observe a cluster with a main-sequence turnoff point at a luminosity of $100L_\odot$. What is the cluster's age, in Myr. What about for a cluster with a turnoff at a luminosity of $10,000L_\odot$?

Exercise 1: A cluster has a main-sequence turnoff at a spectral type $G2$, corresponding to stars of apparent magnitude $m = +10$.
(a) About what is the luminosity, in $L_\odot$, of the stars at the turnoff point?
(b) About what is the age (in Gyr) of the cluster?
(c) About what is the distance (in pc) of the cluster?

[3] Stars just above this main sequence turn-off are dubbed "blue stragglers". They are stars whose close binary companion became a Red Giant with a such big radius that mass from its envelope spilled over onto it. This rejuvenated the mass gainer, making it again a hot, luminous blue star.