9 Inferring Stellar Space Velocities

The next section (§10) will use the inferred orbits of stars in *binary* star systems to directly determine stellar masses. But first, as a basis for interpreting observations of such systems in terms of the orbital velocity of the component stars, let us review the astrometric and spectrometric techniques used to measure the motion of stars through space.

9.1 Transverse speed from proper motion observations

In addition to such periodic motion from binary orbits, stars generally also exhibit some systematic motion relative to the Sun, generally with components both transverse (i.e. perpendicular) to and along (parallel to) the observed line of sight. For nearby stars, the perpendicular movement, called "proper motion", can be observed as a drift in the apparent position in the star relative to the more fixed pattern of more distant, background stars. Even though the associated physical velocities can be quite large, e.g. $V_t \approx 10 - 100$ km/s, the distances to stars is so large that proper motions of stars – measured as an angular drift per unit time, and generally denoted with the symbol μ – are generally no bigger than about $\mu \approx 1$ arcsec/year. But because this is a systematic drift, the longer the star is monitored, the smaller the proper motion that can be detected, down to about $\mu \approx$1 arcsec/century or less for the most well-observed stars.

Figure 9.1 illustrates the proper motion for Barnard's star, which has the highest μ value of any star in the sky. It is so high in fact, that its proper motion can even be followed with a backyard telescope, as was done for this figure. This star is actually tracking along the nearly South-to-North path labeled as the "*Hipparcos*[1] mean" in the figure. The apparent, nearly East-West (EW) wobble is due to the Earth's own motion around the Sun, and indeed provides a measure of the star's parallax, and thus its distance. Referring to the arcsec marker in the lower right, we can estimate the full amplitude of the wobble at a bit more than an arcsec, meaning the parallax[2] is $p \approx 0.55$ arcsec, implying a distance

[1] *Hipparcos* is an orbiting satellite that, because of the absence of the atmospheric blurring, can make very precise "astrometric" measurements of stellar positions, at precisions approaching a milli-arcsec.

[2] given by half the full amplitude, since parallax assumes a 1 au baseline that is half the full diameter of earth's orbit

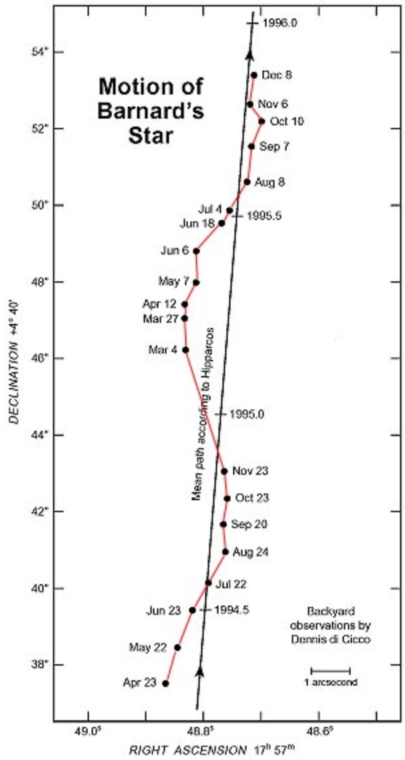

Figure 9.1 Proper motion of Barnard's star. The star is actually tracking along the path labeled as the mean from the *Hipparcos* astrometric satellite. The apparent wobble is due to the parallax from the Earth's own motion around the Sun. Referring to lower right label showing one arcsec, we can estimate the full amplitude of the parallax wobble as about 1.1 arcsec; but since this reflects a baseline of 2 AU from the earth's orbital diameter, the (one-AU) parallax angle is half this, or $p = 0.55$ arcsec, implying a distance of $d = 1/p \approx 1.8$ pc.

of $d \approx 1.8$ pc. By comparison, the roughly South-to-North proper motion has a value $\mu \approx 10$ arcsec/yr.

In general, with a known parallax p in arcsec, and known proper motion μ in arcsec/yr, we can derive the associated transverse velocity V_t across our line of sight,

$$\boxed{V_t = \frac{\mu}{p}\,\text{au/yr} = 4.7\,\frac{\mu}{p}\,\text{km/s}\,,} \tag{9.1}$$

where the last equality uses the fact that the Earth's orbital speed $V_E = 2\pi$ au/yr = 30 km/s. For Barnard's star this works out to give $V_t \approx 90$ km/s, or about 3 times the earth's orbital speed around the Sun. This among the fastest transverse speeds inferred among the nearby stars.

9.2 Radial velocity from Doppler shift

We've seen how we can directly measure the transverse motion of relatively nearby, fast-moving stars in terms of their proper motion. But how might we measure the *radial* velocity component *along* our line of sight? The answer is: via the "Doppler effect", wherein such radial motion leads to an observed shift in the wavelength of the light.

To see how this effect comes about, we need only consider some regular signal with period P_o being emitted from an object moving at a speed V_r toward ($V_r < 0$) or away ($V_r > 0$) from us. Let the signal travel at a speed V_s, where $V_s = c$ for a light wave, but might equally as well be speed of sound if we were to use that as an example. For clarity of language, let us assume the object is moving away, with $V_r > 0$. Then after any given pulse of the signal is emitted, the object moves a distance $V_r P_o$ before emitting the next pulse. Since the pulse still travels at the same speed, this implies it takes the second pulse an extra time

$$\Delta P = \frac{V_r P_o}{V_s} \tag{9.2}$$

to reach us. Thus the period we observe is longer, $P' = P_o + \Delta P$.

For a wave, the wavelength is given by $\lambda = PV_s$, implying then an associated stretch in the observed wavelength

$$\lambda' = P'V_s = (P_o + \Delta P)V_s = (V_s + V_r)P_o = \lambda_o + V_r P_o \, . \tag{9.3}$$

where $\lambda_o = P_o V_s$ is the rest wavelength. The associated relative stretch in wavelength is thus just

$$\frac{\Delta\lambda}{\lambda_o} \equiv \frac{\lambda' - \lambda_o}{\lambda_o} = \frac{V_r}{V_s} \, . \tag{9.4}$$

For sound waves, this formula works in principle as long as $V_r > -V_s$. But if an object moves *toward* us faster than sound ($V_r < -V_s$), then it can basically "overrun" the signal. This leads to strongly compressed sound waves, called "shock waves", which are the basic origin of the sonic boom from a supersonic jet. For some nice animations of this, see

http://www.lon-capa.org/∼mmp/applist/doppler/d.htm

A common example of the Doppler effect in sound is the shift in pitch we hear as the object moves past us. Consider the noise from a car on a highway, for which the "vvvvrrrrrooomm" sound stems from just this shift in pitch from the car engine noise. Figure 9.2 illustrates this for a racing car.

In the case of light $V_s = c$, and so we can define the Doppler shift of light as

$$\boxed{\frac{\Delta\lambda}{\lambda_o} = \frac{V_r}{c} \quad ; \quad |V_r| \ll c \, .} \tag{9.5}$$

This assumes the non-relativistic case that $|V_r| \ll c$, which applies well to most all stellar motions. Straightforward observations of the associated wavelengths of

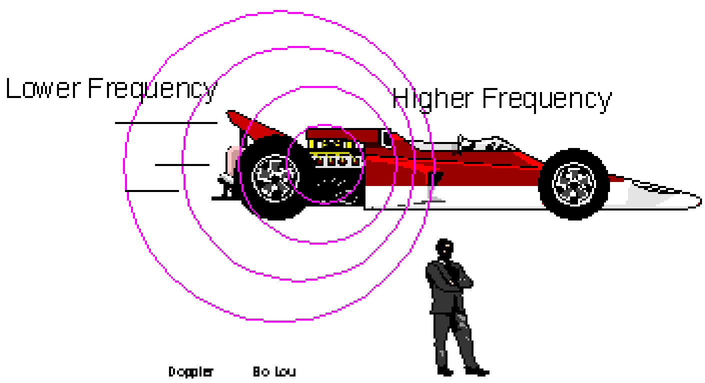

Figure 9.2 Illustration of the Doppler shift of the sound from a racing car.

spectral lines in the star's spectrum relative to their rest (laboratory measured) wavelengths thus gives a direct measurement of the star's motion toward or away from the observer.

For our above example of Barnard's star, observations of the stellar spectrum show a constant *blueshift* of $\Delta\lambda/\lambda = -3.7 \times 10^{-4}$, implying the star is moving *toward* us, with a speed $V_r = zc = -111$ km/s. This allows us to derive the overall *space velocity*,

$$V = \sqrt{V_r^2 + V_t^2}\,. \tag{9.6}$$

For Barnard's star, this gives $V = 143$ km/s, which again is one of the highest space velocities among nearby stars. Mapping the space motion of nearby stars relative to the Sun provides some initial clues about the kinematics of stars in our local region of the Milky Way galaxy.

9.3 Questions and Exercises

Quick Question 1: A star with parallax $p = 0.02$ arcsec is observed over 10 years to have shifted by 2 arcsec from its proper motion. Compute the star's tangential space velocity V_t, in km/s.

Quick Question 2: For the star in QQ#1, a line with rest wavelength $\lambda_o = 600.00$ nm is observed to be at a wavelength $\lambda = 600.09$ nm.

a. Is the star moving toward us or away from us?
b. What is the star's Doppler shift z?
c. What is the star's radial velocity V_r, in km/s?
d. What is the star's total *space* velocity V_{tot}, in km/s?