

# **3. Tools for Solar Observations-I**

Solar telescopes. Resolution, MTF, seeing. High resolution telescopes. Spectrographs.

# Types of Solar Observations

- Electro-magnetic observations (optical, UV, X-rays, -rays).
  - Filtergrams
  - Spectrograms
  - Dopplergrams
  - Magnetograms
- Special Observations
  - Neutrino Experiments
  - Sound Waves (Helioseismology)
  - Particles (in situ measurements)
  - Gravitational Waves

# Limitations

- Photon noise.
- Seeing.
- Diffraction limit

## Photon noise

The photon noise is caused by random fluctuations in the photon flux emitted by a steady thermal source. Although photons are expected to arrive at a constant average rate of  $r$  photons per unit time interval they are not expected to arrive at regular intervals.

At any particular interval of time  $\tau$  the expected number of photons is  $\langle N \rangle = r\tau$ , but the observed number will not be exactly this. The probability to detect exactly  $N$  photons is:

$$P(N) = \exp(-r\tau) \frac{(r\tau)^N}{N!}.$$

This is called Poisson distribution. The important property of this distribution is that the standard deviation is equal to the square root of the mean:

$$\sigma = \sqrt{\langle N \rangle}.$$

The signal-to-noise ratio is:

$$SNR = \frac{N}{\sqrt{N}} = \sqrt{N}.$$

## Example

Consider observations with a 60-cm telescope of an area on the Sun of  $0.3 \times 0.3 \text{ arcsec}^2$  with exposure 1 ms, spectral bandwidth  $10 \text{ m}\text{\AA}$  at  $\lambda = 5000 \text{ \AA}$  and 1% efficiency of the whole system (telescope-spectrograph-detector).

The number of photons radiated by the Sun per unit time per unit area per frequency interval per steradian can be estimated by using the Planck Law:

$$N(\lambda) = \frac{B_\nu}{h\nu} = \frac{2\nu^2}{c^2} \frac{1}{e^{h\nu/kT} - 1}.$$

For  $T \approx 6000 \text{ K}$ ,  $\nu = c/\lambda \approx 6 \times 10^{14} \text{ Hz}$ :

$$N \approx 4.3 \times 10^6 \text{ photon s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sterad}^{-1}.$$

The frequency bandwidth is:  $\Delta\nu = \nu\Delta\lambda/\lambda \approx 1.2 \times 10^9 \text{ Hz}$ .

The radiating area on the Sun is:

$$S = (0.3 \cdot 7.25 \times 10^7)^2 = 4.7 \times 10^{14} \text{ cm}^2.$$

The angular size of the receiving area of the telescope is:

$$A = \frac{\pi(d/2)^2}{R^2},$$

where  $d = 60$  cm is the telescope diameter,  $R = 1.5 \times 10^{13}$  cm is the distance from the Sun. Then,  $A \approx 1.26 \times 10^{-23}$  sterad.

Finally, the number of photons received by the telescope (with 1% efficiency) is:

$$N_T = N \times S \times \Delta t \times \Delta \nu \times A \approx 304 \text{ photons.}$$

The photon noise is  $1/\sqrt{N_T} \approx 1/17 \approx 5.7\%$ .

The SNR can be increased by increasing the frequency bandwidth, the exposure time, the observed area on the Sun or the telescope diameter.

## Seeing

The Sun heats the Earth atmosphere and causes turbulence that leads to fluctuation of the refraction index. The typical frequency of the image fluctuation is up to 100 Hz. If the exposure time is less than 0.01 sec than the seeing effects are reduced.

Solar telescope are often made very tall to reduce the turbulence effects and improve seeing.



Mount Wilson Observatory 150-foot tower



Big Bear Solar Observatory

# Point Spread Function

Consider a real image in the x-y plane. At each point the intensity  $I(x, y)$  has contributions which belong to neighboring points,  $\xi, \eta$ , of the ‘perfect’ image,  $I_0$ . This is described by the point spread function  $PSF(x, y; \xi, \eta)$ .

$$I(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_0(\xi, \eta) PSF(x, y; \xi, \eta) d\xi d\eta.$$

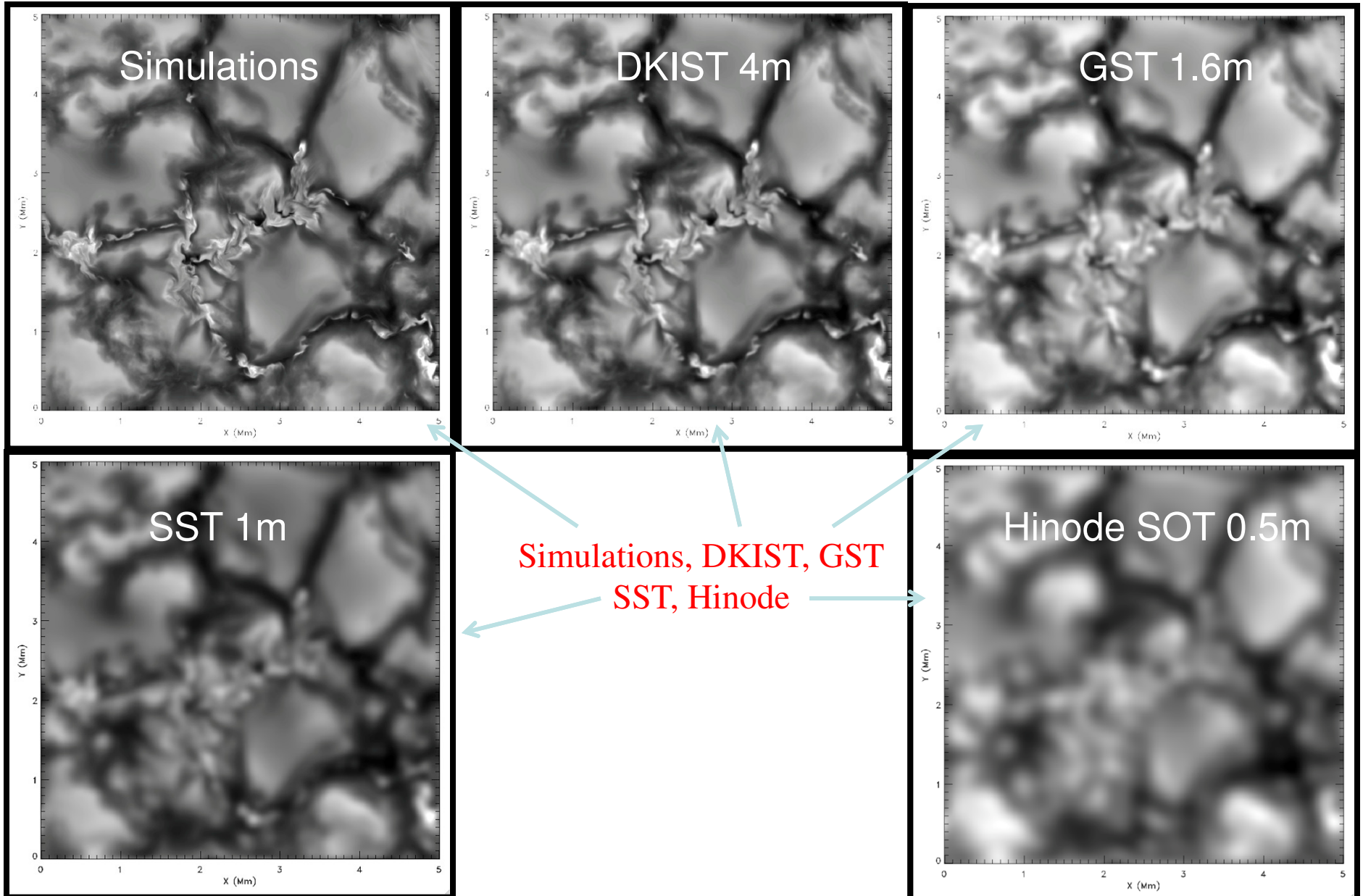
The convolution can be written as:

$$I = I_0 * PSF.$$

In a simple case, the PSF depends only on  $\xi - x$  and  $\eta - y$ .

Then, we use the convolution theorem: each Fourier component of  $I$  is a product of the corresponding Fourier component of  $I_0$  and Fourier component of  $PSF$ .

# Effect of Telescope Point-Spread Function



**The Fourier transform of PSF is called the Modulation Transfer Function (MTF).**

$$MTF(k_x, k_y) = \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} PSF(x, y) \exp[-2\pi i(k_x x + k_y y)] dx dy \right|,$$

where  $k_x$  and  $k_y$  are the spatial wavenumbers.

The advantage of using MTF is that several degrading effects can be taken into account by simply multiplying MTF functions.

For instance, if  $MTF_{\text{telescope}}$  describes the optical imperfection of the telescope then in the combination with seeing the total MTF is:

$$MTF_{\text{total}} = MTF_{\text{telescope}} \cdot MTF_{\text{seeing}} \cdot$$

# Fourier Relationship between MTF and PSF

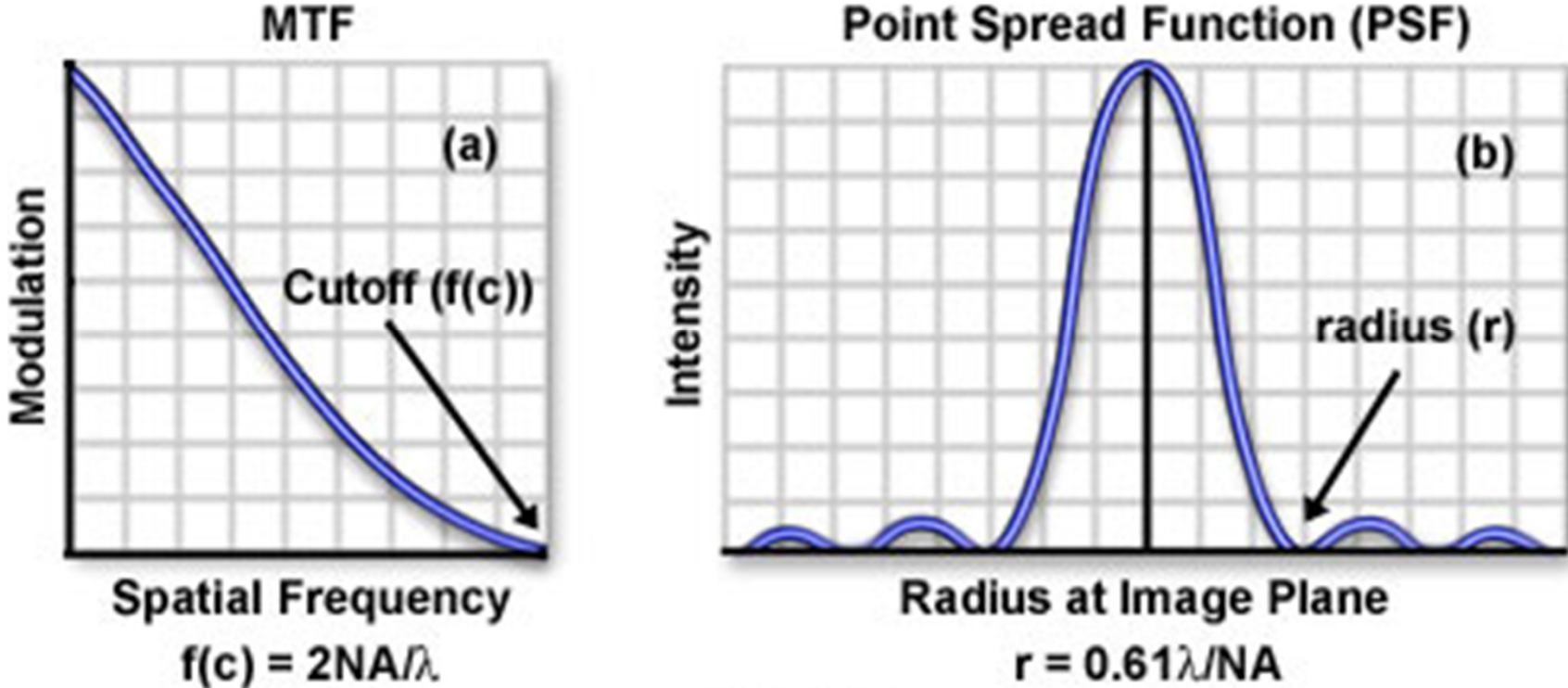


Figure 3

Numerical Aperture:  $NA = D/2f$  (or  $N = f/D$ : f-number")  
D is the telescope aperture  
f is the focal length

## Diffraction Limited Telescopes.

Diffraction contributes to the spatial resolution of telescopes. It can be described for large focal lengths  $f$  by the following PSF:

$$PSF(r) = \frac{J_1(br)}{br}, \quad b = \frac{\pi D}{\lambda f},$$

where  $D$  is the aperture of the telescope,  $\lambda$  is the wavelength,  $J_1$  is the Bessel function. The Bessel function has 0 at  $br_1 \approx 3.832$ . Then,

$$r_1 = \frac{3.832}{b} = \frac{3.823\lambda f}{\pi D}.$$

This is the radius of the ‘central Airy disc’.

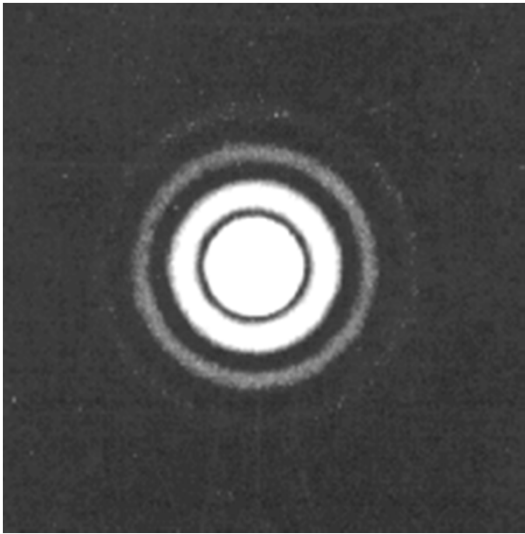
The angle,  $\alpha = r_1/f$ , corresponds to **the angular resolution of the telescope (in radians)**:

$$\alpha = \frac{3.823}{\pi} \frac{\lambda}{D}.$$

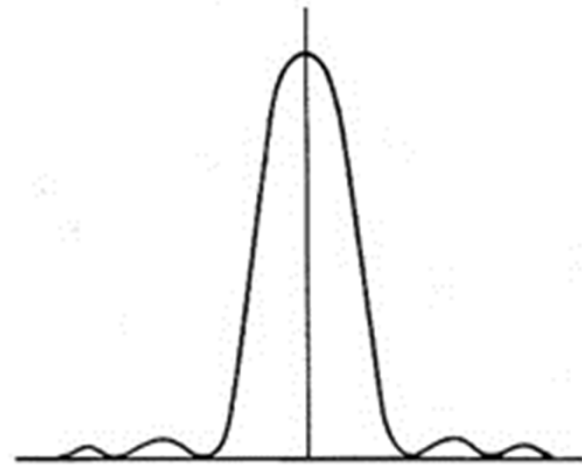
The corresponding MTF is:

$$MTF_D(k) = \frac{2}{\pi} \left[ \arccos(k/k_m) - \frac{k}{k_m} \sqrt{1 - (k/k_m)^2} \right],$$

where  $k_m = b/\pi$  is the largest wave number transmitted by the telescope;  $MTF_D = 0$  for  $k \geq k_m$ .

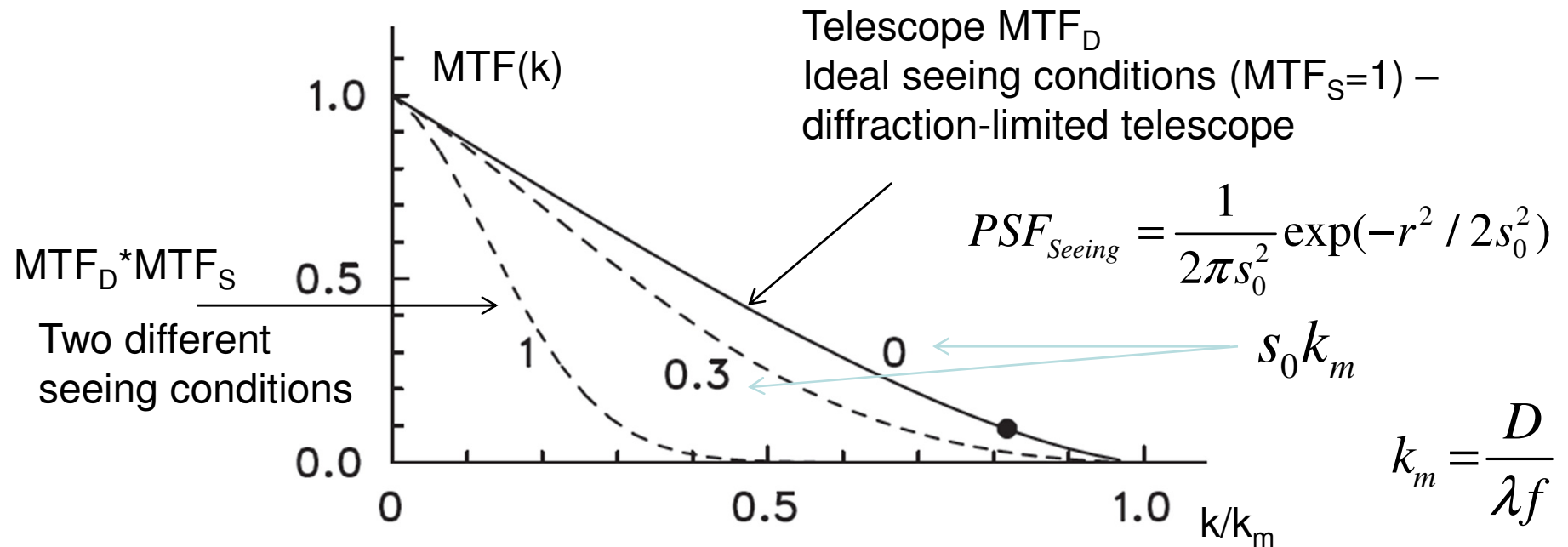


Airy disk



Light transmission curve for an Airy disk

# Modulation Transfer Function (MTF) vs spatial frequency $k$



The solid curve of shows MTF. According to its definition, the modulation transfer function is the factor by which a signal is reduced at each spatial frequency  $k$ . The spatial frequency corresponding to the “spatial resolution”  $r_1$  is  $k = 1/r_1 = k_m/1.22$ , and  $MTF_D = 0.0894$  for this  $k$  (the black dot in). This means that of the original, say, 15% intensity contrast of the solar granulation, little more than 1% is left in the image if a telescope is used which according to the criterion just “resolves” the granular structure.

**Exercise:** What aperture needs a diffraction-limited solar telescope in order to transmit, at  $\lambda = 500$  nm, 60% of the contrast of a 1 arcsec sharp feature?

Solution:

From the MTF plot: MTF=0.6 at  $k/k_m=0.32$

$$k_m = b / \pi, \quad b = \frac{\pi D}{\lambda f}$$

$$k = 0.32k_m = 0.32 \frac{b}{\pi} = 0.32 \frac{D}{\lambda f} \quad \Rightarrow \quad D = \frac{\lambda f k}{0.32}$$

$$\text{angular resolution } \alpha = \frac{r_1}{f} = \frac{1}{kf}$$

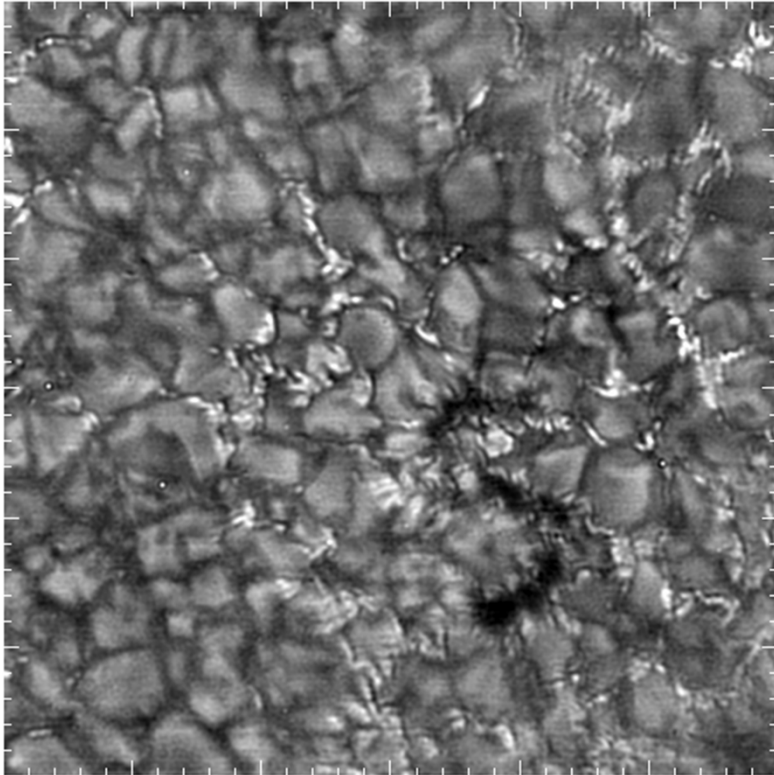
$$D = \frac{\lambda}{0.32\alpha} = \frac{500 \times 10^{-9}}{0.32 \cdot 4.848 \times 10^{-6}} = 0.322 \text{ m.}$$

## Exercises:

- 1) Calculate the aperture and the focal length of the HMI instrument for Solar Dynamics Observatory. This instrument obtains full-disk images of the Sun in Fe I 6173Å line using a  $5 \times 5 \text{ cm}^2$  CCD detector with  $4096 \times 4096$  pixels. The angular diameter of the Sun is 32 arcmin.
- 2) Calculate the angular resolution (in arcsec) of the 4 m DKIST Solar Telescope for observations of the H-alpha line. Determine the size of the smallest solar features that can be seen with this telescope.
- 3) What aperture needs a diffraction-limited solar telescope in order to transmit, at  $\lambda = 6563 \text{ \AA}$ , 50% of the contrast of a 0.1 arcsec sharp feature?

## **Adaptive Optics.**

High resolution observations of the Sun have become increasingly important for solving many of the outstanding problems in solar physics. Near diffraction limited snapshot images taken for example at the 76 cm R.B. Dunn Solar Telescope (DST) at Sacramento Peak show an amazing amount of fine structure.



**High resolution g-band (430.5 nm) image obtained at the DST/SP using frame selection. Exposure time 8 ms. Tick marks are 1". The bright points mark the sites of small magnetic field elements on the surface of the Sun. The FWHM of the smallest g-band bright point observed is 0".13**

Figure 1 shows a g-band (430.5 nm) image taken with a very short exposure time of 8 ms. In this image small magnetic flux elements are visible as bright points.

Images of this quality are rare. More importantly, in order to study the physics of these flux elements, or solar fine structure in general, spectroscopy and polarimetry of the fine structure is required. The exposure times are then typically of order 1 second and the resolution currently achieved in spectroscopic/polarimetric data on a more or less regular basis is typically 1 arcsec, which is insufficient to effectively study solar fine structure.

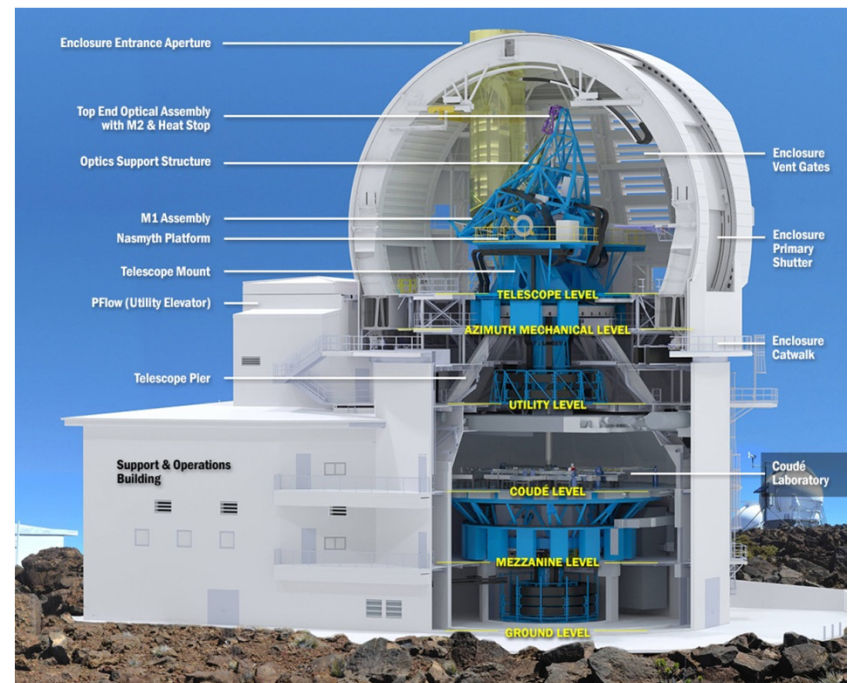
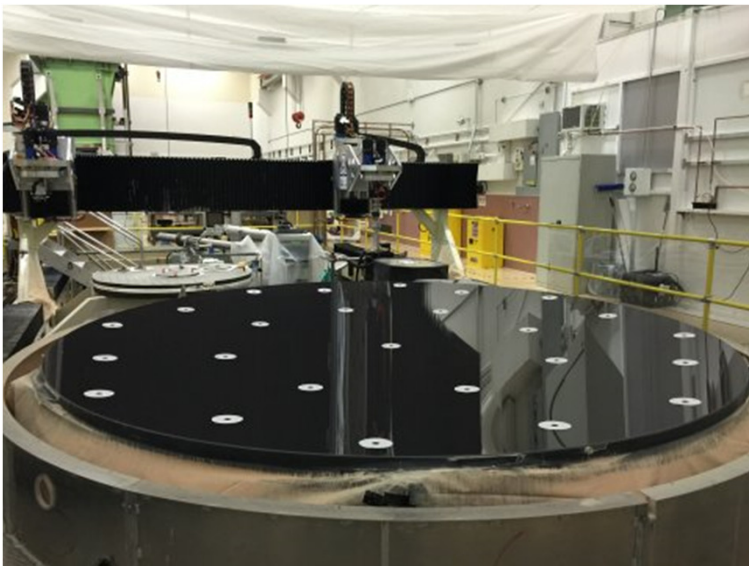
Further motivation for the development of solar adaptive optics is provided by theoretical model predictions. Sophisticated simulations of magneto-hydrodynamic processes predict structure below the resolution limit of current solar telescopes, which have apertures of order 1 meter. These theoretical predictions are re-enforced by polarimetric measurements, which provide a filling factor (the ratio of magnetic to non-magnetic area) and thus give an indirect measurement of the size of the magnetic elements present on the solar surface.

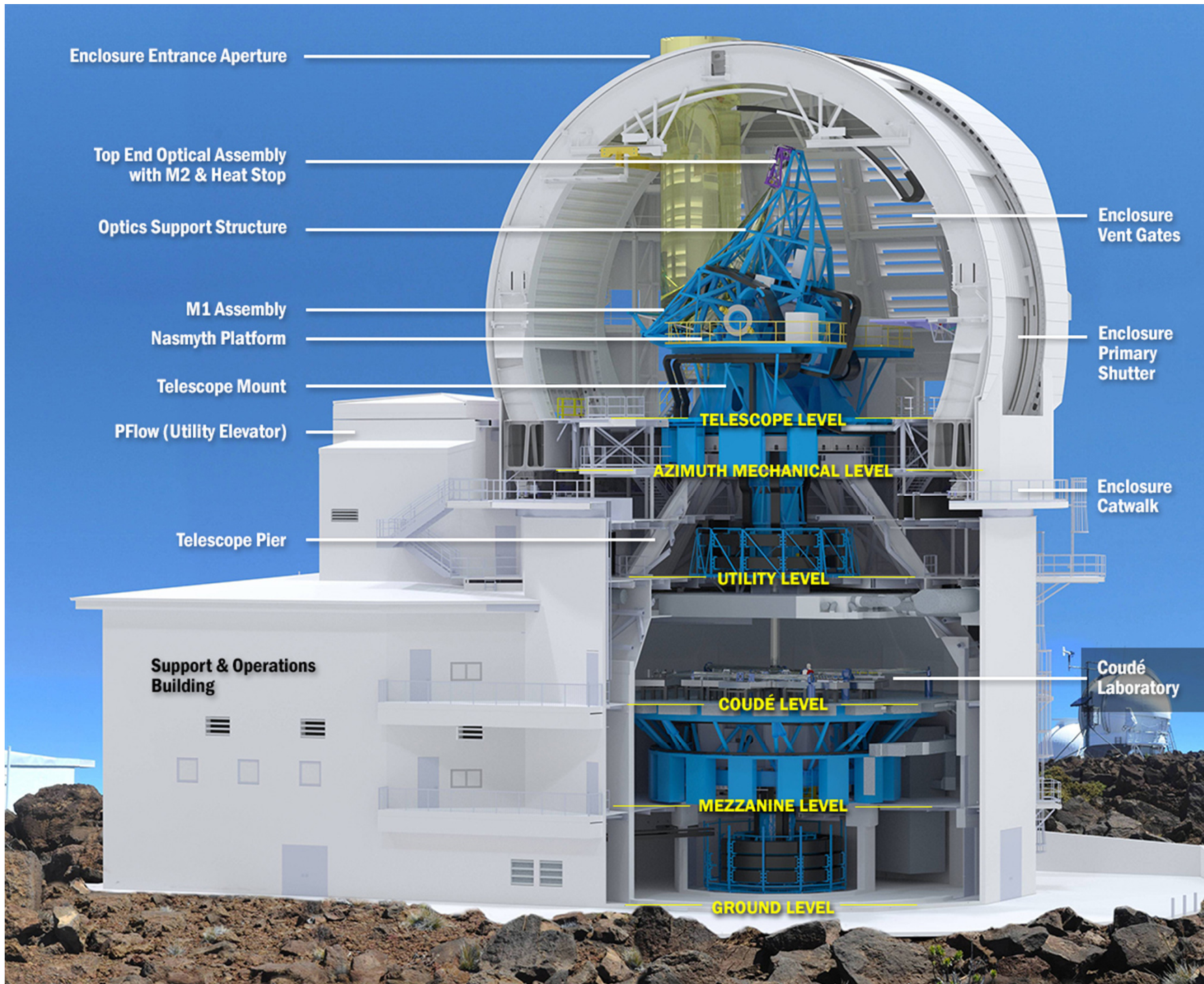
Observations of solar structures below the resolution limit of about 0."2 of current solar telescopes are needed to study the important physical processes that occur on such small scales.

A new generation solar telescope with a substantially increased aperture is needed. Solar telescope with a 4m aperture is under construction.

Adaptive optics is essential to achieve consistent spatial resolution of 0."1 or better from ground based observatories.

<http://dkist.nso.edu/>





# Advanced Technology Solar Telescope

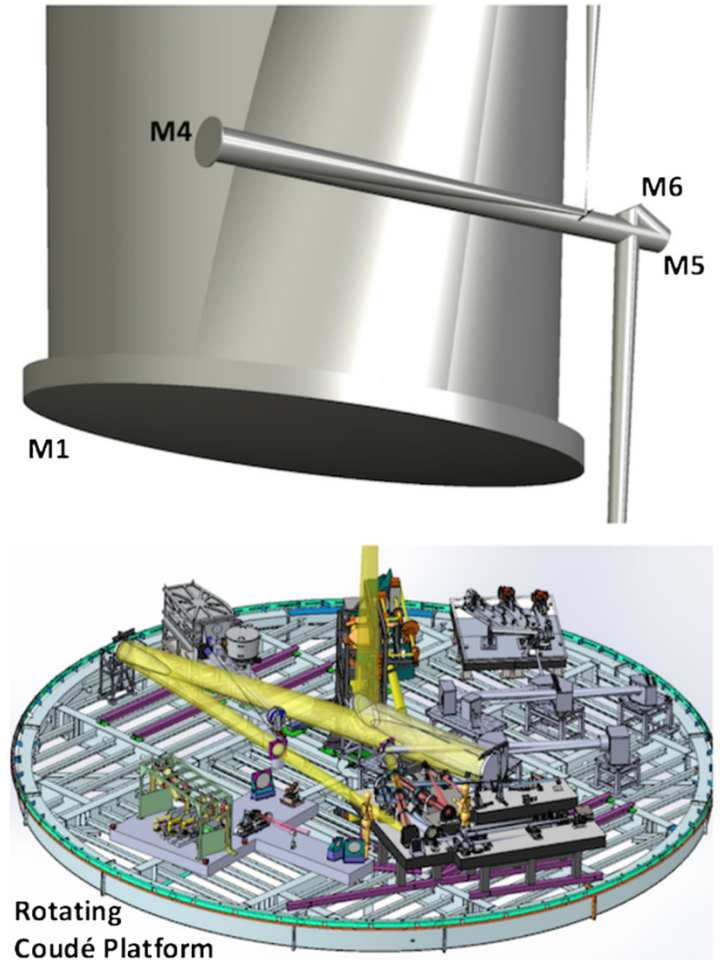
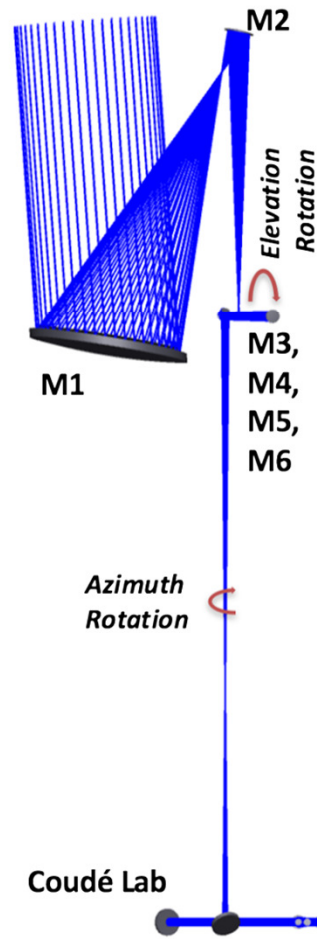
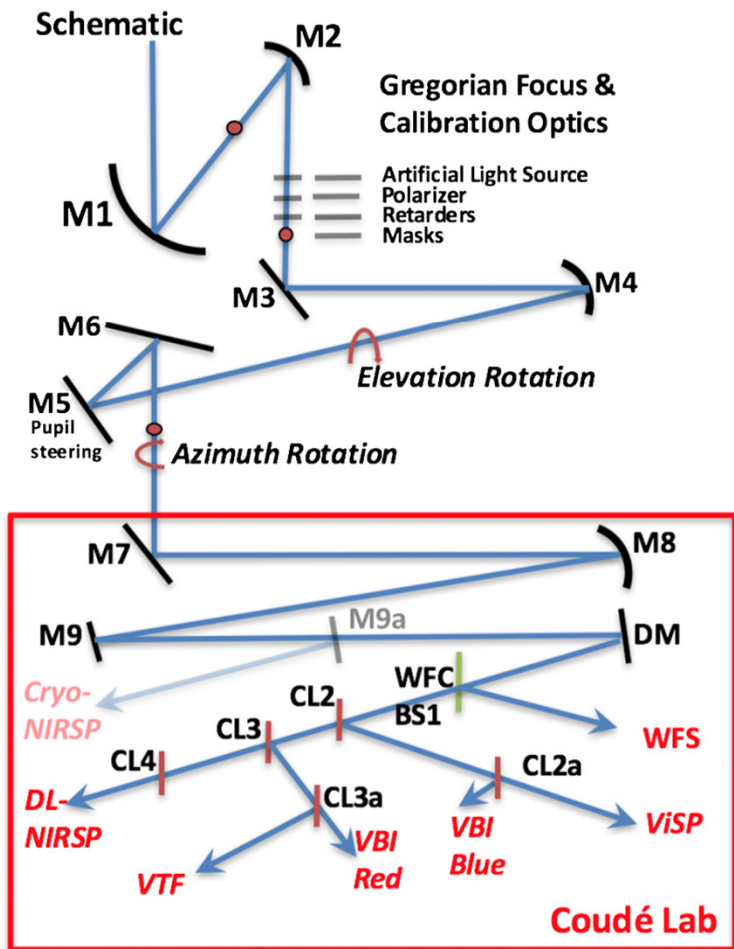
## Daniel K. Inouye Solar Telescope

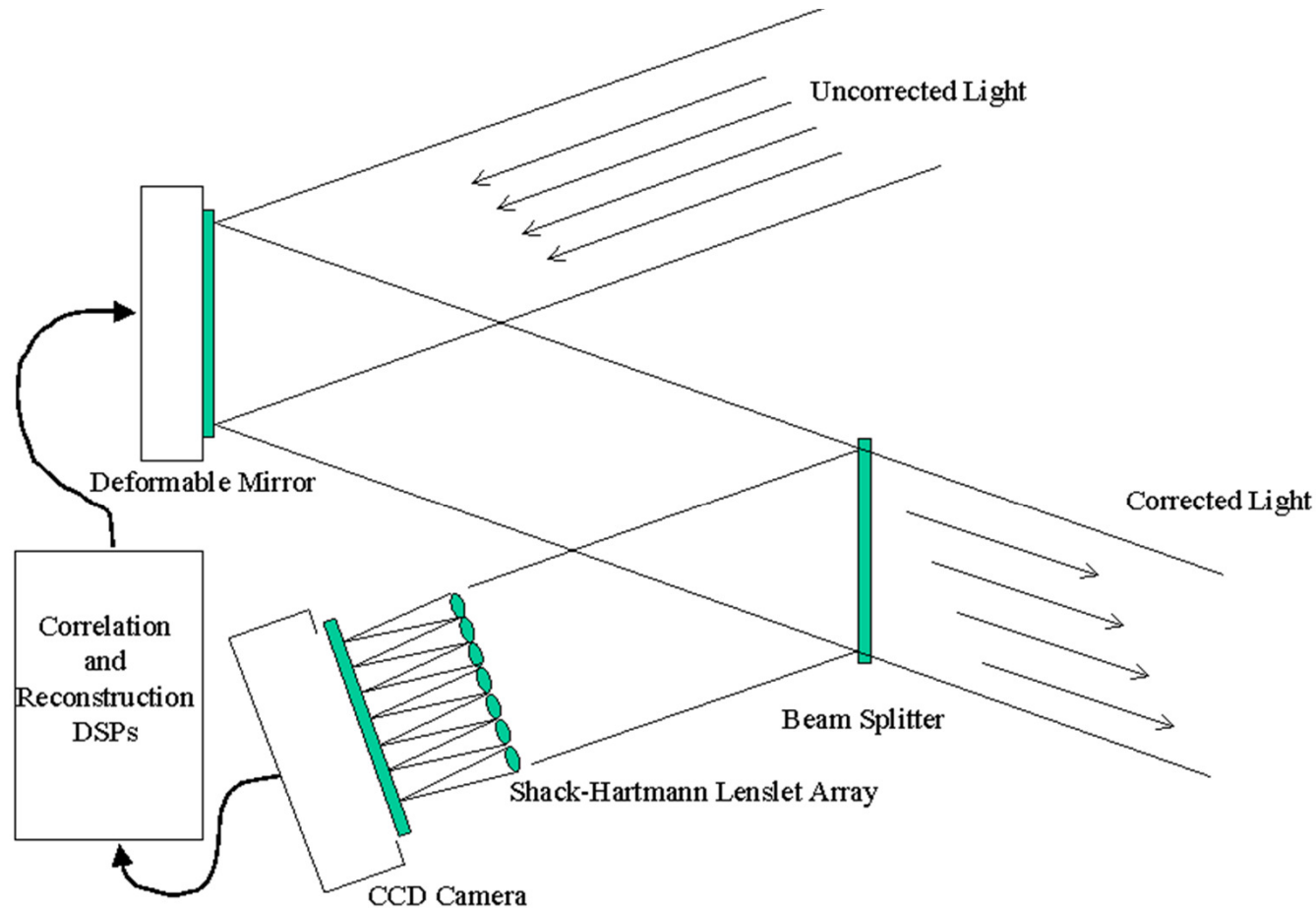
Table 3.1 *Summary of Basic Telescope and Performance Parameters*

<b>Aperture:</b>	4m
<b>Optical Configuration:</b>	Gregorian, off-axis
<b>FOV:</b>	5 arcmin
<b>Optical Quality:</b>	<0."1 over FOV
<b>Adaptive Optics:</b>	Strehl >0.5 within isoplanatic patch
<b>Wavelength Coverage:</b>	350 nm - 35 microns
<b>Polarization Accuracy:</b>	>10E – 4
<b>Coronagraphic:</b>	in the IR
<b>Scattered Light:</b>	<10E – 5 at $r/r_{\text{sun}} = 1.1$ and $>1\mu$

The Strehl ratio (SR or ‘Strehl’) is a fraction of flux in the diffraction limited core of the point spread function under average atmospheric conditions.

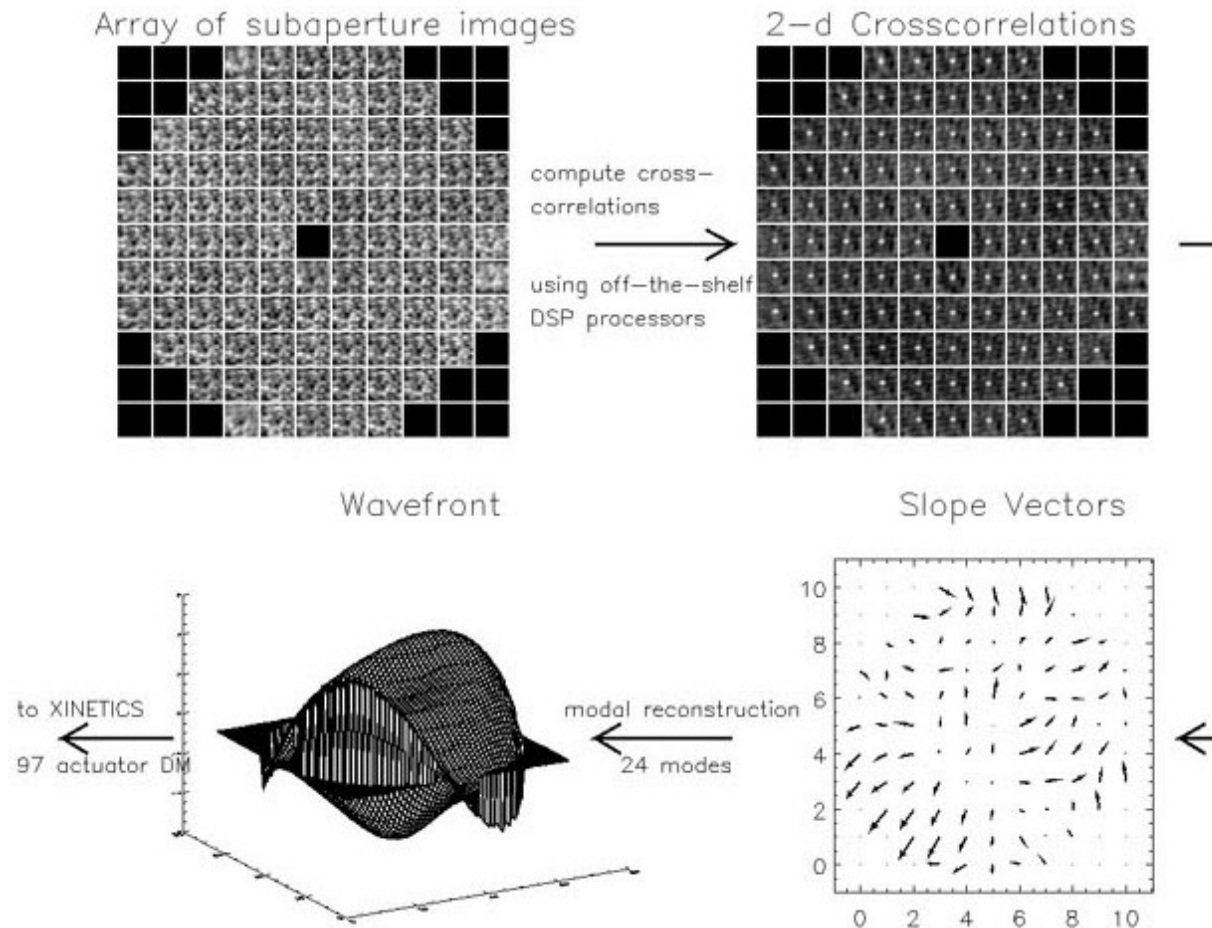
# DKIST optics





### **The NSO adaptive optics system.**

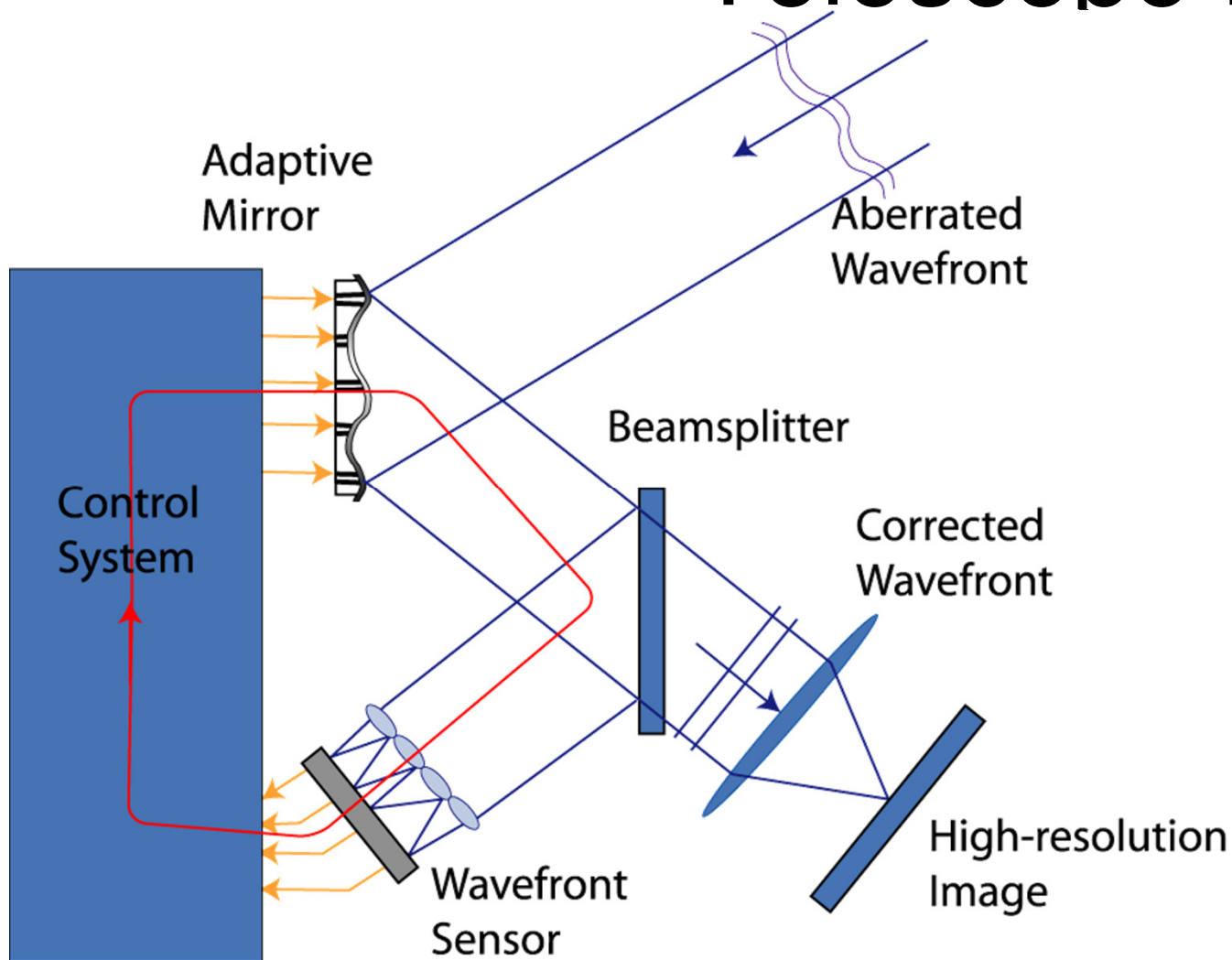
Since a point source is in general not available on the solar surface the wavefront sensor has to be able to sense the wavefront using solar granulation, a low-contrast extended source, as target. The system is based on a correlating so-called Shack-Hartmann system, which uses arbitrary scenes, including solar granulation, as targets for wavefront sensing.



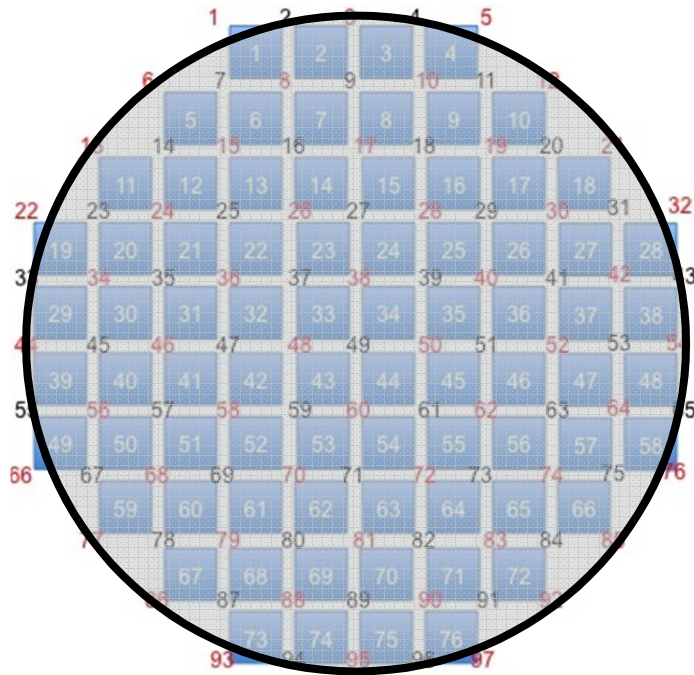
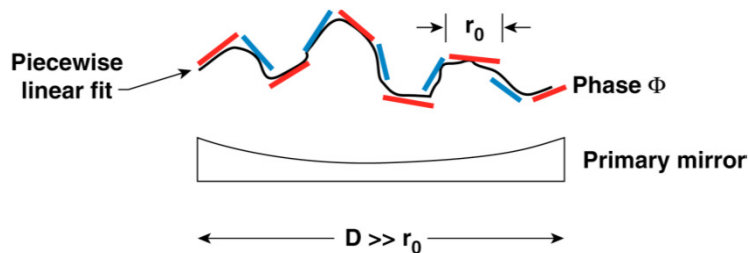
Principle of correlating Shack-Hartmann wavefront sensor working on granulation. Top Left: Granulation images formed by a lenslet-array. In this example the aperture (76 cm) was sampled with 11 square subapertures resulting in a subaperture diameter of 7cm. Top Right: Cross-correlations derived from images referenced a particular subaperture. Bottom Right: Shift vectors derived from Cross-Correlations. Global tilt is removed from the tilt vector map. Bottom Left: Wavefront derived from tilt map using a modal reconstruction scheme.

The telescope aperture is sampled by an array of lenslets, which in turn forms an array of images of the object. In this case the object is granulation. The subaperture size is 7 cm and diffraction limits the rms contrast of the granulation images to 2.5-3%, depending on the seeing conditions, compared to 6-8% when imaged through the full aperture of the telescope. Nevertheless, the image contrast is sufficient to compute cross-correlations between subaperture-images and a selected subaperture-image, which serves as reference.

# Adaptive Optics at 1.6m Telescope BBSO



# 1<sup>st</sup> Generation AO: AO-76

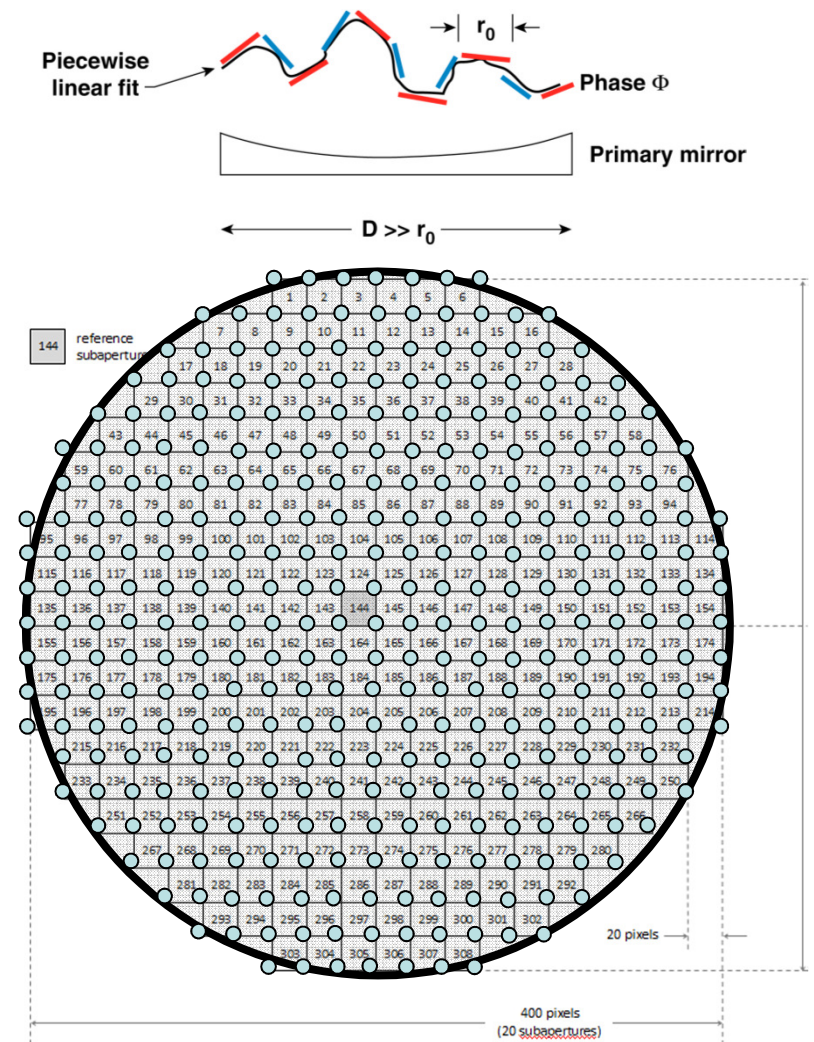


AO-76

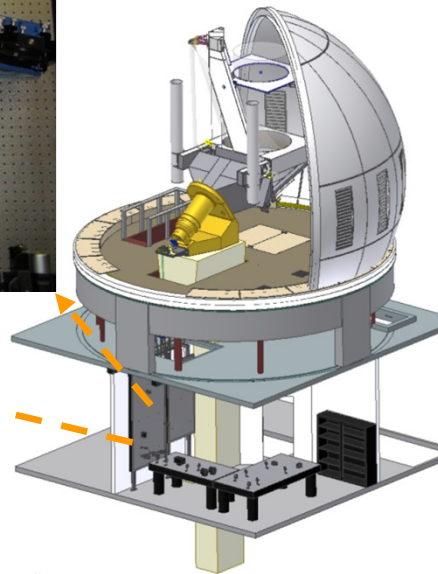
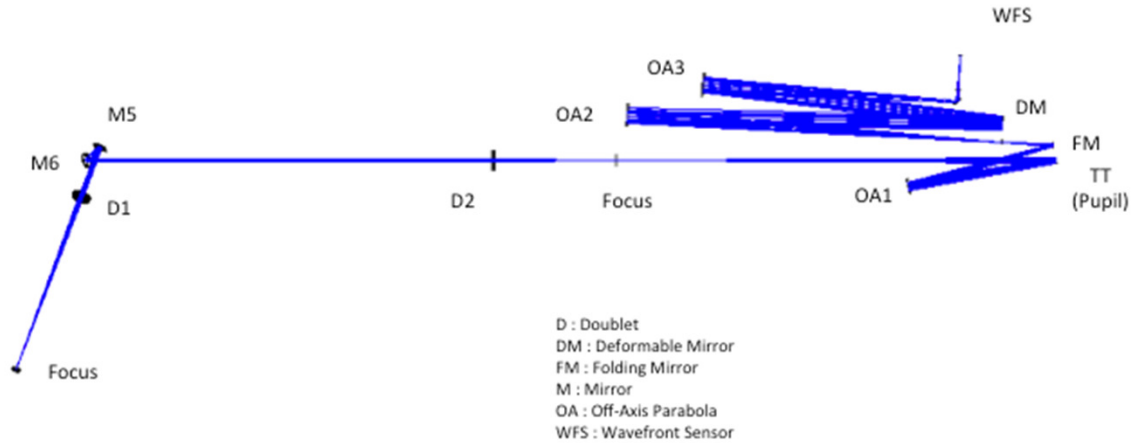
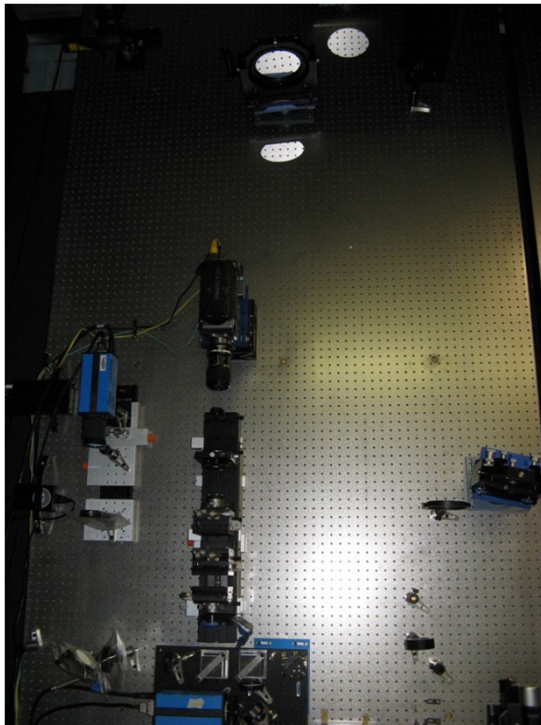
- ❖ Shack Hartmann WFS with 76 sub-apertures (AO-76): 10 sub-apertures across the NST PM
- ❖ Xinetics Deformable Mirror with 97 actuators and 7 mm spacing
- ❖ Baja camera with a frame rate of 2500 Hz for 200 by 200 sub-array
- ❖ Bitware Hammerhead Boards with 40 digital signal processors (DSPs)
- ❖ Closed-loop Bandwidth: 120 Hz
- ❖ Strehl ratio: 0.7 in the NIR under median BBSO seeing
- ❖ BBSO  $r_0 = 6 \sim 8$  cm @ 500 nm, NST requires 20 sub-apertures across pupil

# 2<sup>nd</sup> Generation AO: AO-308

- ❖ AO-308 is operational, AO-308 is a collaboration between BBSO and NSO
- ❖ Shack Hartmann WFS with 308 sub-apertures (AO-308): 20 sub-apertures across the NST primary mirror
- ❖ Xinetics Deformable Mirror with 357 actuators and 5 mm spacing
- ❖ Phantom V.7.3 camera with a frame rate of 2000 Hz for 400 by 400 pixel sub-array
- ❖ Bitware TigerSHARC Boards with 16 digital signal processors (DSPs)
- ❖ Closed-loop Bandwidth: 120 Hz
- ❖ Acquire diffraction limited imaging over the telescope's full range of operation



# AO-308 Layout and GUI



**AO308**  
File View Help

**Subapertures Image**

**Histogram**

Equalization  
 16,384  
 Range : 1,455 ~ 7,421  
 X and Y : 373, 372  
 Value : 0  
 Index : 171  
 Shifts (x, y) : -0.32, 0.64  
 Max Abs : 2.14  
 Rolloffcount : 0  
 Display Shifts  
 High Gain  
 Average 1000

**Information**

GUI AO308.EXE (2012.08.30.)  
 DSP AC0308acq1a.DXE (2012.08.30.)  
 DSP AC0308acq1b.DXE (2012.08.30.)  
 DSP AC0308acq2a.DXE (2012.08.30.)  
 DSP AC0308acq2b.DXE (2012.08.30.)  
 DSP AC0308\_B3.DXE (2012.08.23.)

Mask Subapertures  
 Display Zoom

Total Frames : 1,846,000  
 Total Time [sec] : 1,000  
 Average Rate [fps] : 1,843.84  
 Current Rate [fps] : 1,885.12  
 Data Transfer Time [ms] : 1.76  
 Display Rate [fps] : 3.76

Shift Algorithms :  SAD  XCorr

**Control On/Off**

Off All  
 TT On  
 DM On  
 TT and DM On

**Reference Subaperture**

Xerr :  
 Yerr :  
 RMS : 1,245.14

Auto Update Reference  
 XY tolerance : 0.00  
 RMS tolerance : 0.00  
 Interval [sec] : 1.00

**DM Actuators**

Max Volt : 1.99  
 Min Volt : -1.97  
 Index : 116  
 Abs. Volt : -0.41  
 Rel. Volt : 0.02  
 Flat Volt : -0.43

Voltage Display :  
 Input  
 Output

**Tip/Tilt Actuators**

X : - +  
 Y : - +  
 Abs. Volt : 15.69, 11.49  
 Rel. Volt : 1.69, -2.51  
 Offset Volt : 14.00, 14.00

**Zernike Polynomial**

By Slope  By Voltage

RMS (1-43)	0.043
01 Piston	0.000
02 Tilt X	-0.020
03 Tilt Y	-0.014
04 Defocus	0.079
05 Astigmatism X	0.119
06 Astigmatism Y	-0.223
07 Coma X	0.025
08 Coma Y	0.005
09 Trefoil X	-0.035
10 Trefoil Y	0.041
11 P. Spherical	-0.008
12 S. Astigmatism X	0.019
13 S. Astigmatism Y	-0.014

**Log Messages :**

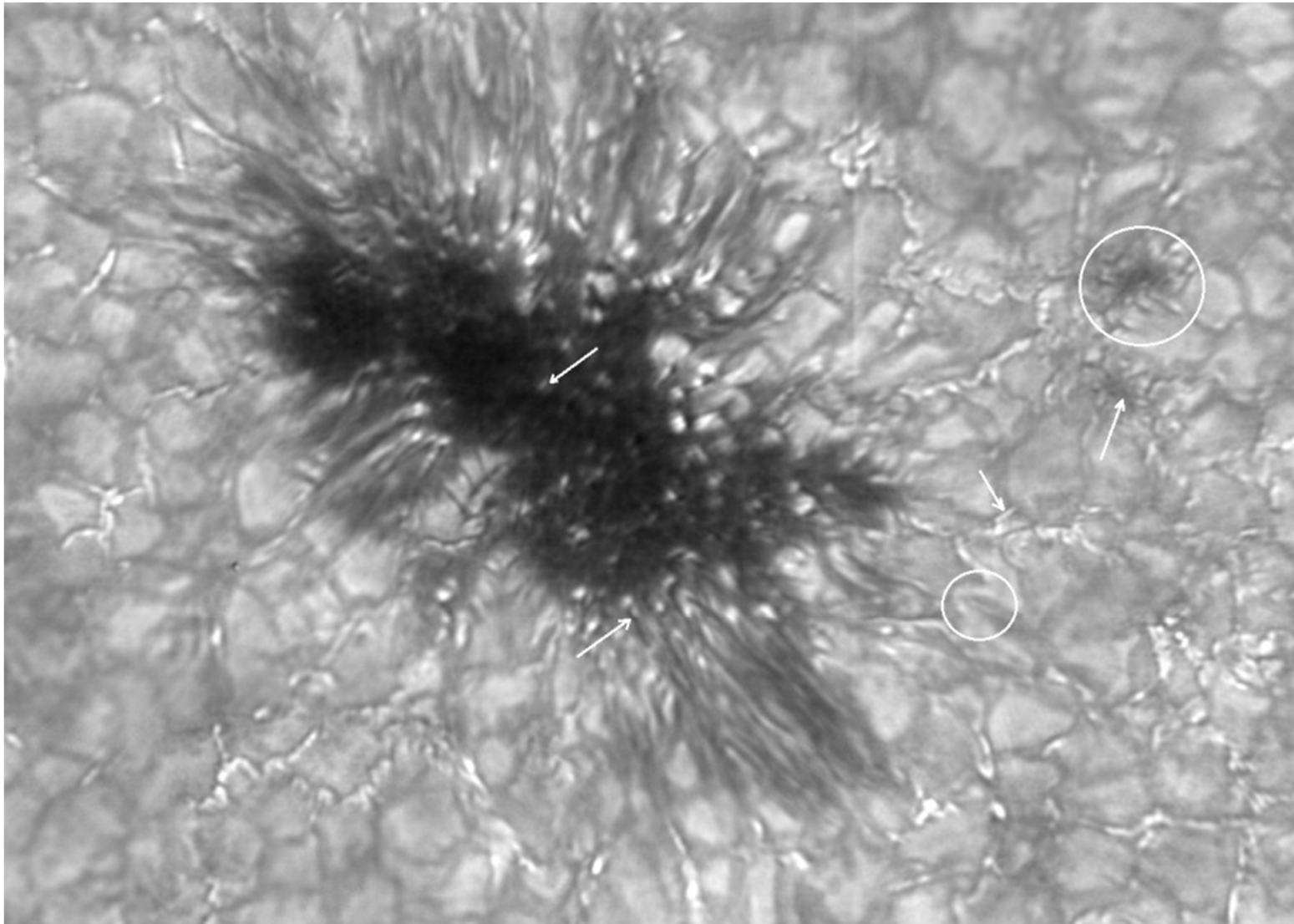
```

11:13:35 SetEnableTilt(1)
11:13:35 SetEnableDM(0)
11:13:32 WriteRef() success.
11:13:31 SetEnableTilt(0)
11:13:31 SetEnableDM(0)
11:12:53 begin frame = 1,761,500
11:12:47 SetEnableTilt(1)
11:12:47 SetEnableDM(0)
11:12:30 SetEnableTilt(0)
11:12:30 SetEnableDM(0)
11:12:27 SetEnableTilt(1)
11:12:27 SetEnableDM(1)
11:12:23 SetEnableTilt(1)
11:12:23 SetEnableDM(0)
11:12:22 WriteRef() success.
11:12:20 WriteGains()
11:12:20 WriteGains()
11:12:14 SetEnableTilt(0)
  
```

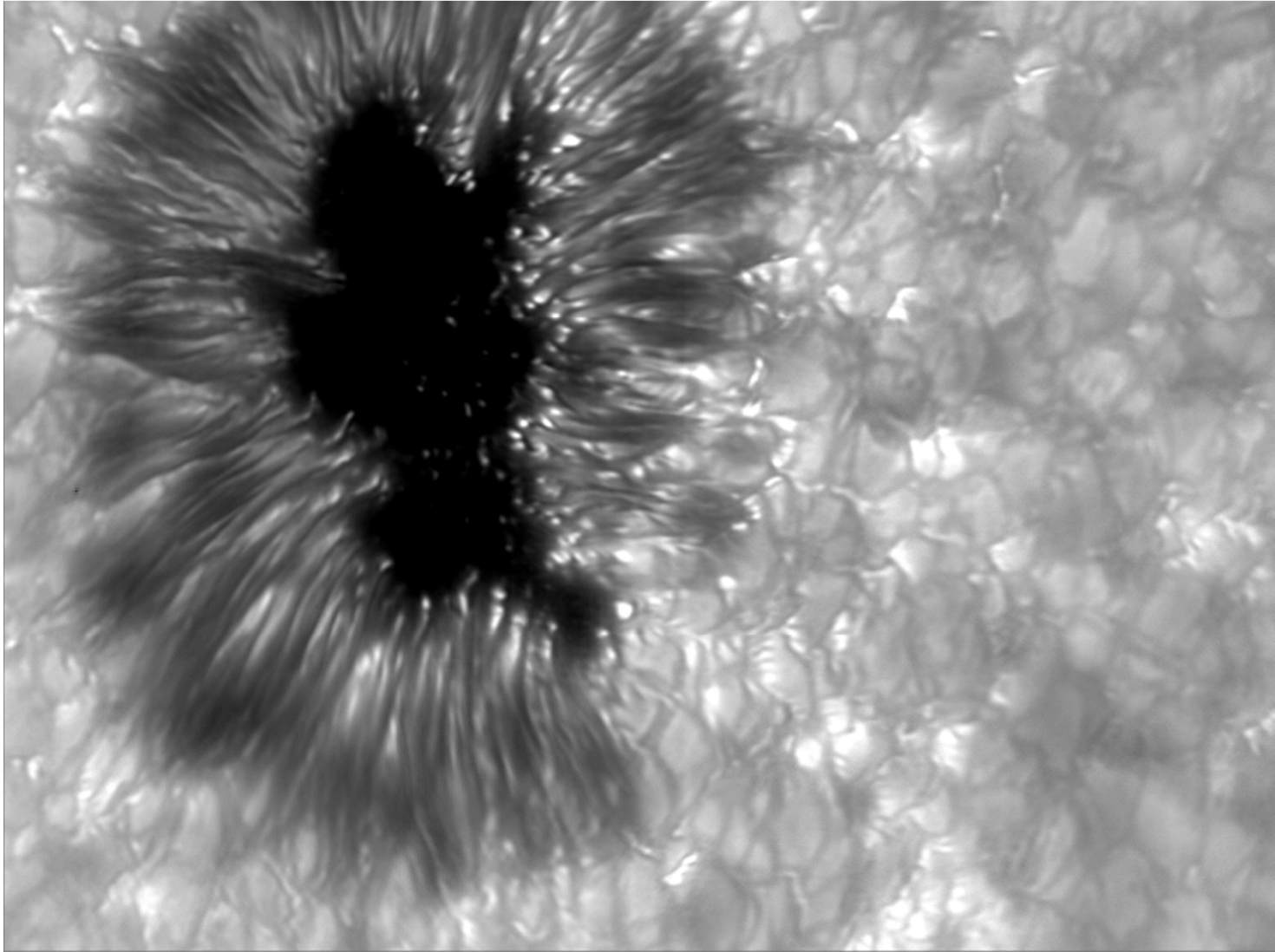
**Setup**

On  Off  
 Subaperture Images  
 Shift (Subapertures)  
 Volt (Actuators)  
 Continuous Shifts

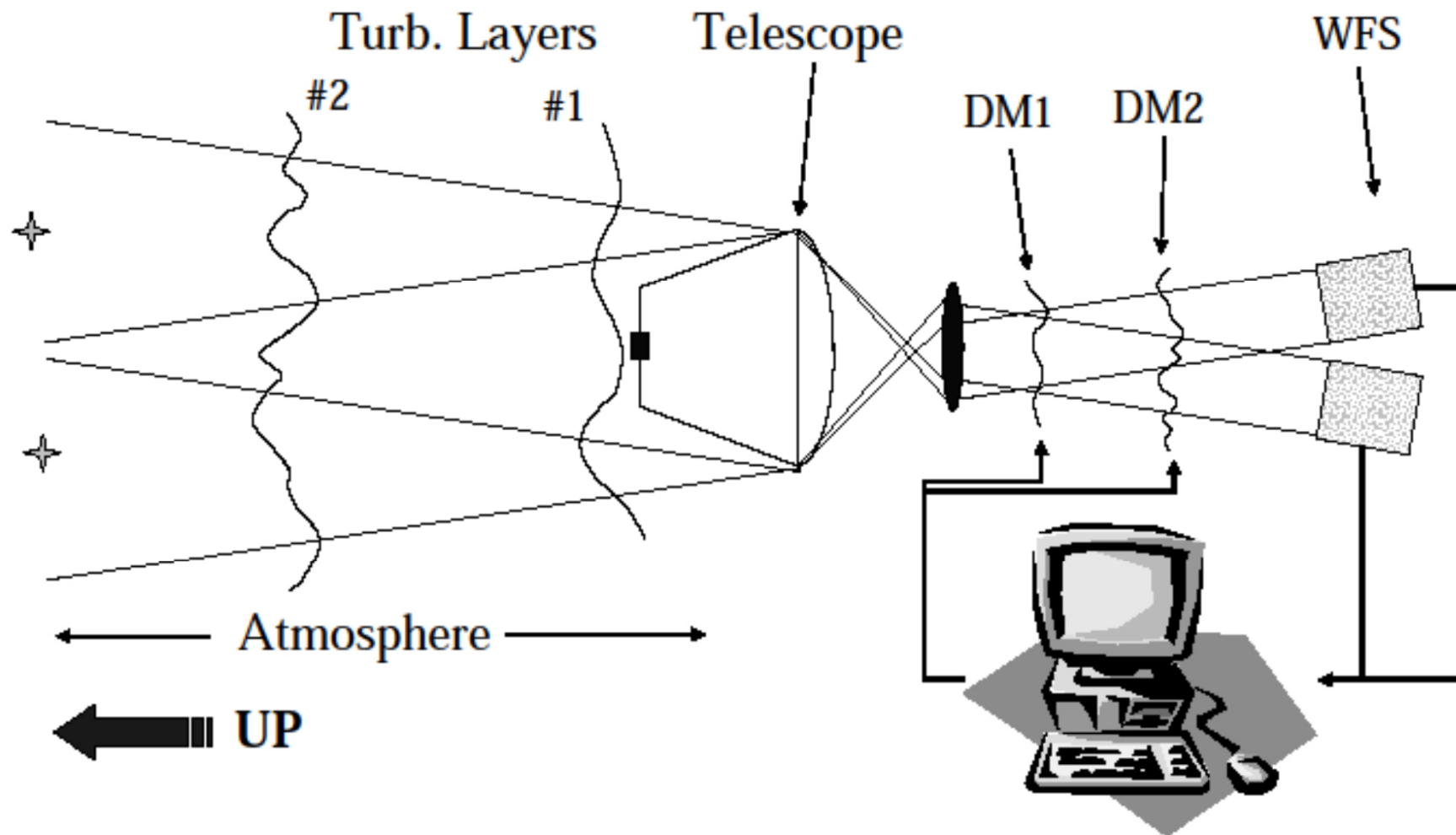
# G-band with AO-308 (9/04/12)



# G-band with AO-308 (9/06/12)

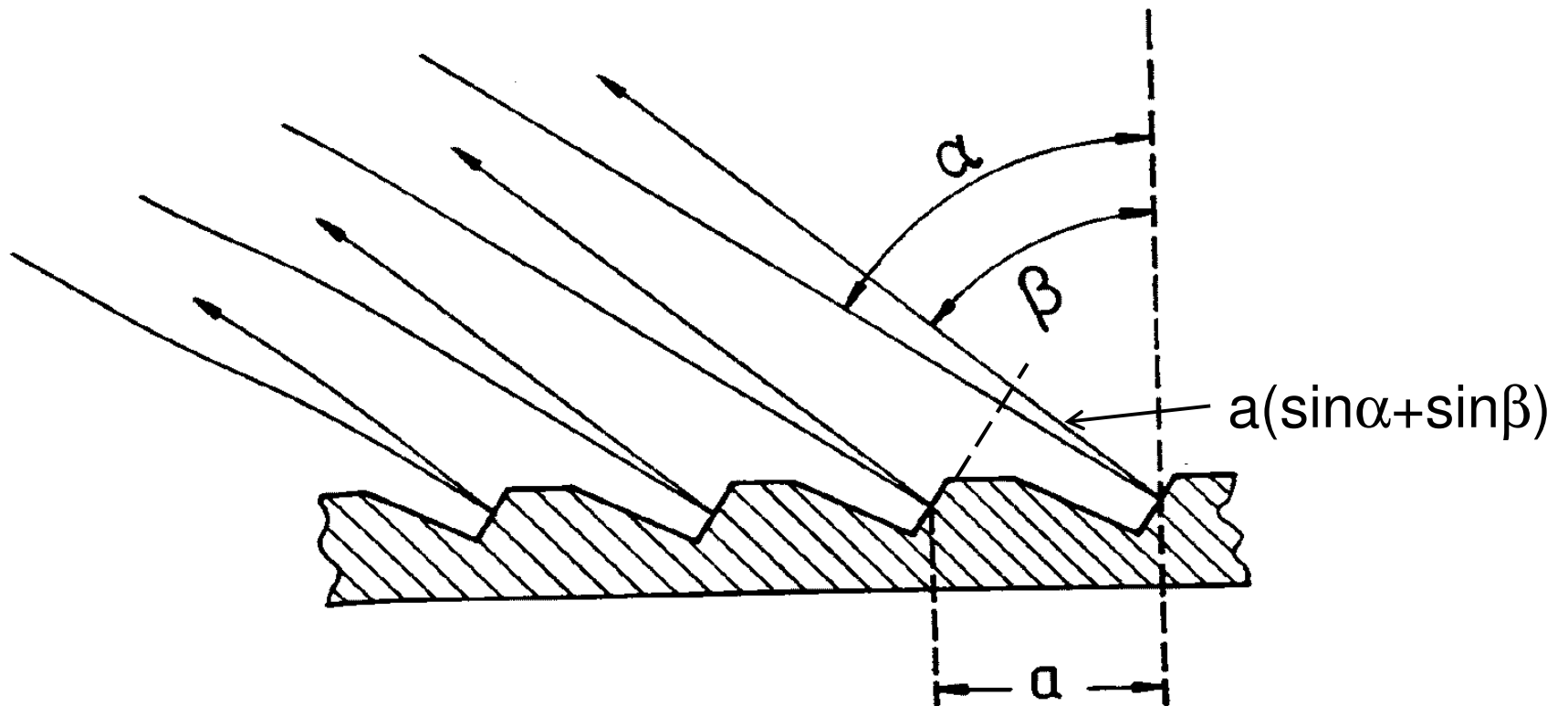


# Third Generation AO: Multi-Conjugate AO in Collaboration with NSO/SP & KIS



# Spectrograph

Most solar spectrographs use reflection gratings.



**Blazed reflection grating (echelle grating).**

Consider a reflection grating with a distance between grooves,  $a$ , ('grating constant'). If  $\alpha$  is an angle of incidence, and  $\beta(\lambda)$  the angle of diffraction, then the directions of the maximum intensity,  $\beta$ , are given by the 'grating equation':

$$m\lambda = a(\sin \alpha + \sin \beta),$$

where  $m$  is the order of the spectrum.

Differentiation of this equation gives the angular dispersion:

$$\frac{d\beta}{d\lambda} = \frac{m}{a \cos \beta}.$$

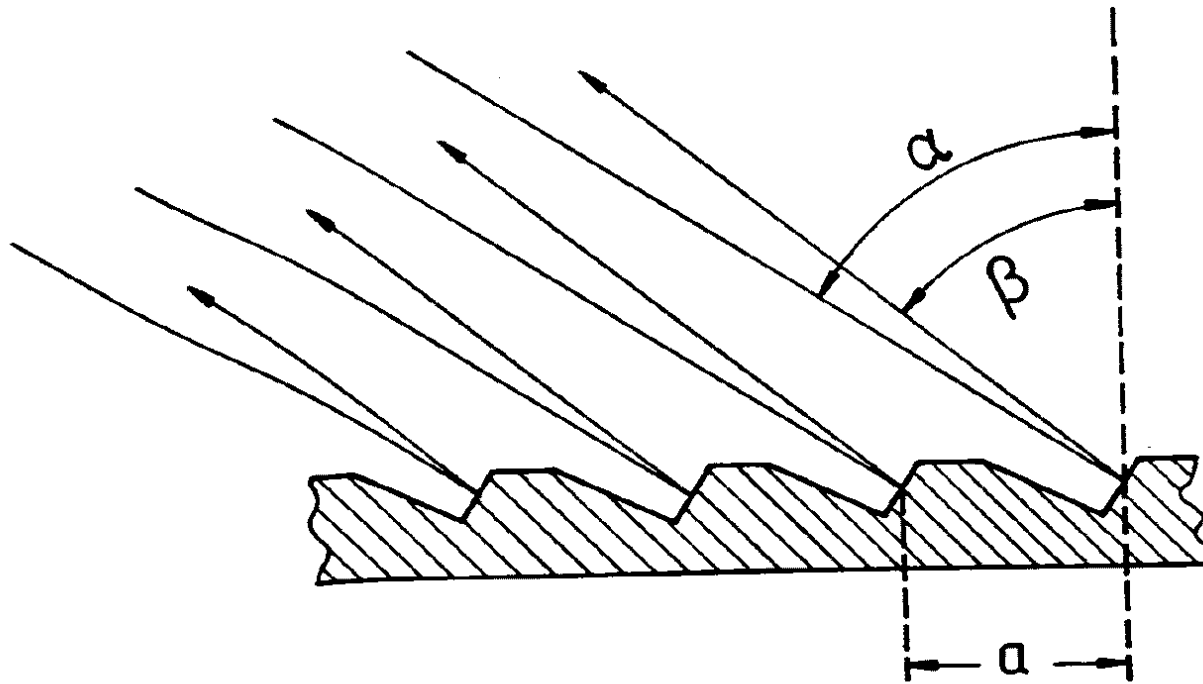
The linear dispersion in the focal plane,  $f$ , is

$$\frac{dx}{d\lambda} = f \frac{d\beta}{d\lambda}.$$

For  $\beta \approx \alpha$ ,

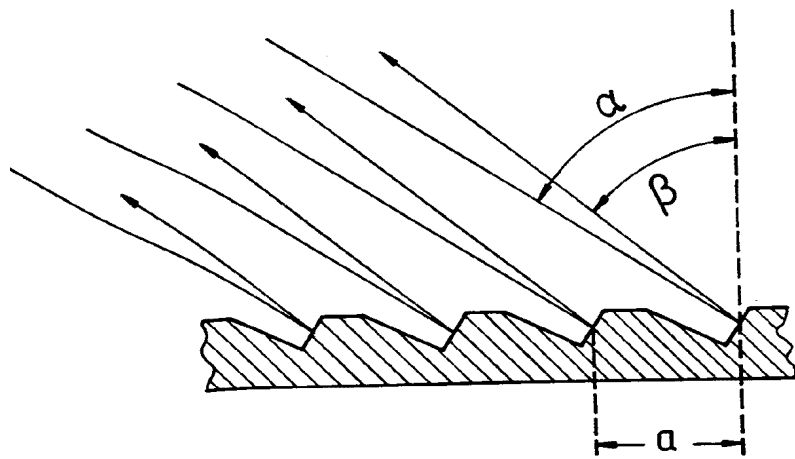
$$f = \frac{dx}{d\lambda} \frac{a \cos \alpha}{m}.$$

A typical grating has 600 grooves/mm, and typically,  $m = 5$ ,  $\alpha = 60^\circ$ . If we want to have a resolution  $dx/d\lambda = 6 \text{ mm}/\text{\AA}$ , then the focal length,  $f \approx 9.5 \text{ m}$ . From this we can determine the aperture of the telescope.



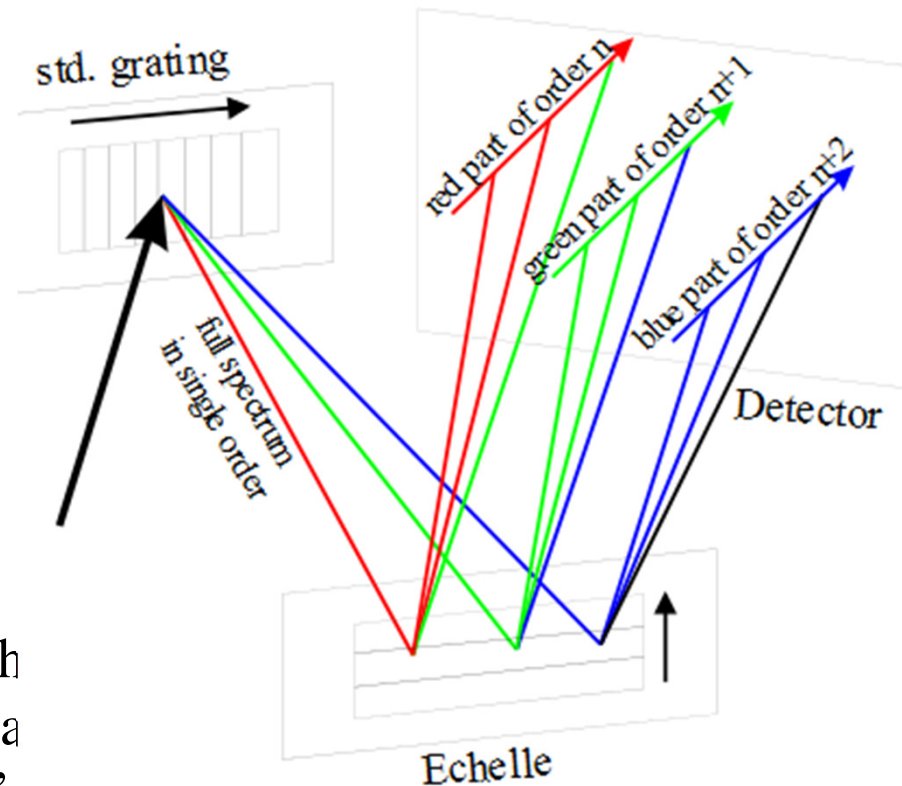
Echelle gratings allow to work with high orders  $m \approx 50$ . They have a special profile, strongly 'blazed', and have high reflectivity. The spectra for various order  $m$  may overlap, and this is used to record simultaneously several lines with a single camera. Unwanted parts are separated by masks and filters.

# Echelle grating



Echelle gratings allow to work with high orders  $m \approx 50$ . They have a special profile, strongly 'blazed', and have high reflectivity. The spectra for various order  $m$  may overlap, and this is used to record simultaneously several lines with a single camera. Unwanted parts are separated by masks and filters.

# Echelle spectrometer



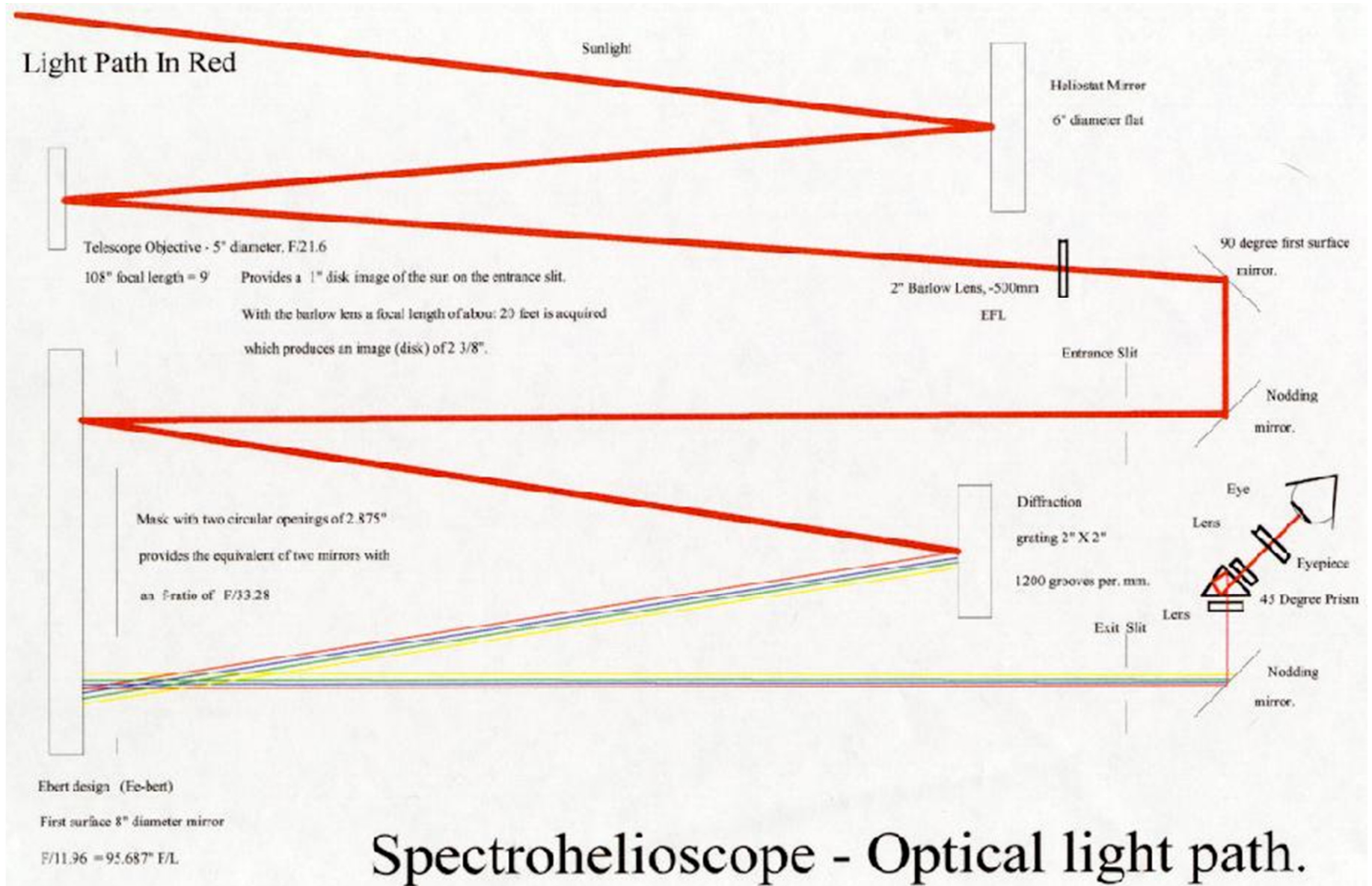
The first standard grating is optimized for a single lower order. The echelle is mounted orthogonally in such a way that the highly illuminated orders of the echelle are transversally separated. Different parts of the spectrum are recorded simultaneously.

## **Spectrohelioscope.org**

The instrument is built by Leonard Higgins in Sonoma county. "The instrument is a pleasure to use, and provides many different aspects and challenges in regard to observing the closest star, our sun."



[Spectrohelioscope.org](http://Spectrohelioscope.org)

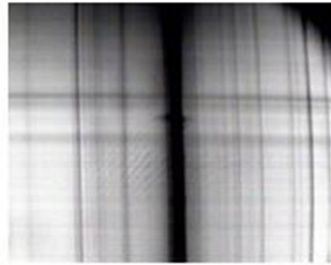


**The optical scheme. Telescope Objective 5',  $f = 108''$ , diffraction grating  $2'' \times 2''$ , 1200 grooves per mm.**



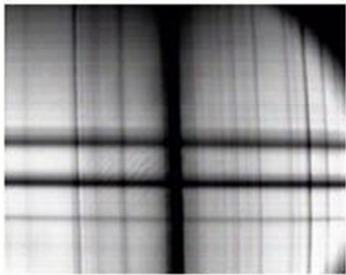
Horizontal lines are due to sunspots

In the deep red beyond the B line.

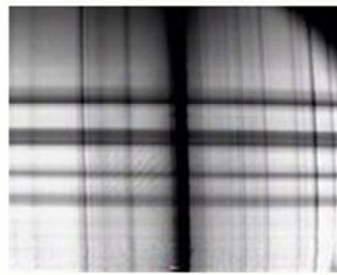


H-alpha line very apparent  
Notice the spike on either side - possible filament.

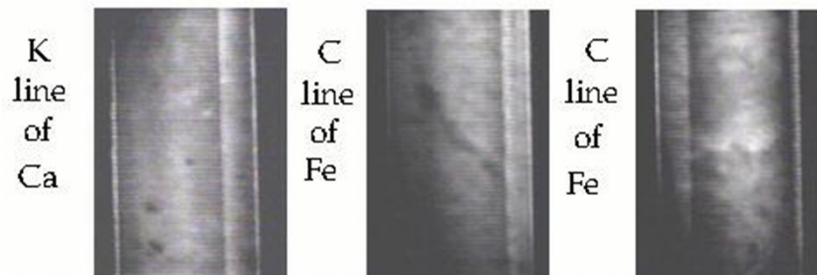
Solar spectra taken on June 27, 1999.



You can see the umbra and penumbra of the sunspot.



Many sunspots and much detail to observe.



Views of the sun's disk with mirror synthesizer operating.

**Exercise.** Estimate the spectral resolution of Leonard Higgins's spectrohelioscope.