7. Internal structure. II

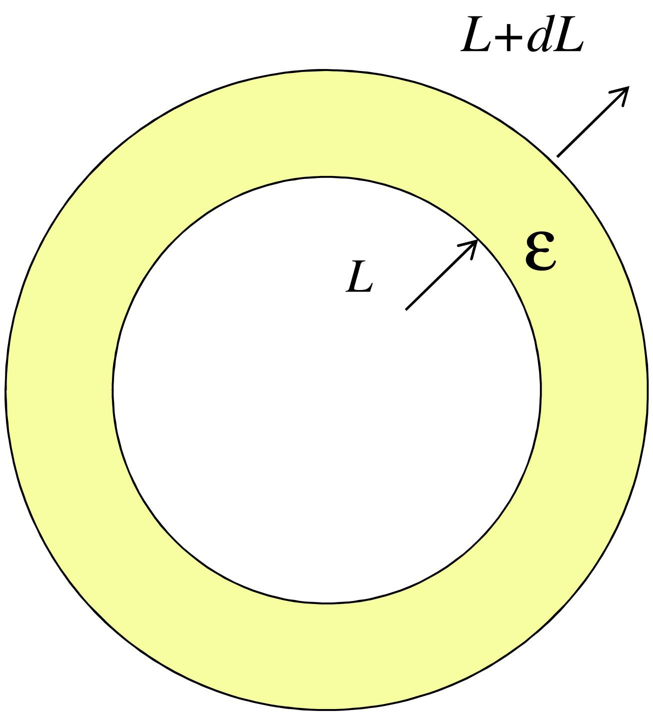
Internal Structure II.

1. Equations of solar evolution
2. Scaling laws
3. Nuclear reactions: p-p chain
4. Convective energy transport
5. Solar neutrino problem

Energy transfer and balance equations

The total energy flux, $L = 4\pi r^2 F$, integrated over a sphere of radius r :

$$L = -\frac{16\pi acT^3}{3\kappa\rho}\frac{dT}{dr}.$$



If ε is the energy release per unit mass then the energy flux change in a shell *dr* is:

$$dL = \varepsilon\rho 4\pi r^2 dr$$

$$\frac{dL}{dr} = 4\pi\rho r^2\varepsilon$$

This is the equation for conservation of energy (energy balance).

Equations of the stellar structure

$$\frac{dm}{dr} = 4\pi\rho r^2 \qquad (1)$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \qquad (2)$$

$$\frac{dL}{dr} = 4\pi\rho r^2 \varepsilon \qquad (3)$$

$$\frac{dT}{dr} = -\frac{\kappa\rho}{16\pi r^2 acT^3} L \equiv -F \qquad (4)$$

$$P = \frac{\Re\rho T}{\mu} \qquad (5)$$

$$\mu = \frac{1}{2X + \frac{3}{4}Y + \frac{1}{2}Z} \qquad (6)$$

$$\varepsilon = \varepsilon_0 X^2 \rho T^4 \qquad (7)$$

$$\kappa = \kappa_0 (X+1) Z \rho T^{-3.5} \qquad (8)$$

Kramer's opacity law

These equations describe the structure of stellar radiative zones. In the convection zone Eq.(4) is replaced by an equation of convective energy transport, e.g. mixing length theory.

A numerical code for solving these equations is available in the book: C.J. Hansen, S.D. Kawaler, *Stellar Interiors. Physical Principles, Structure and Evolution,* Springer, 1995.

Scaling Laws

Simple estimates can be obtained without solving the equations for solar structure numerically.
Temperature inside the Sun can be roughly estimated from the equation of hydrostatic balance and the equation of state.

Using Eq.(1-2) and (4) we obtain the following relations:

$$\left.\begin{aligned} \frac{dm}{dr} &= 4\pi\rho r^2 \\ \frac{dP}{dr} &= -\frac{Gm\rho}{r^2} \\ P &= \frac{\Re\rho T}{\mu} \end{aligned}\right\} \qquad \begin{aligned} \rho &\sim \frac{M}{R^3} \\ \frac{P}{R} &\sim \frac{GM^2}{R^5} \\ P &\sim \frac{\Re\rho T}{\mu_0}. \end{aligned}$$

The molecular weight for $X \sim 0.7$, $Y \sim 0.28$, and $Z \sim 0.02$ is: $\mu_0 \sim 0.6$.

Then, $T \sim \dfrac{GM\mu_0}{R\Re} \sim \dfrac{6.67\times10^{-8}\cdot 2\times10^{33}\cdot 0.6}{8.31\times10^{7}\cdot 7\times10^{10}} \sim 1.4\times10^{7}\,\mathrm{K}.$ (9)

More general relations among the global properties of stars can be represented in terms of scaling laws.

Consider homologous transformations: $m = M\tilde{m}(r/R)$, $\rho = \rho_0 \tilde{\rho}(r/R)$ etc, where M , R , ρ_0 etc are scaling factors.

Then Eq.(1)-(8) require the scaling factors to satisfy the relations:

$$\frac{dm}{dr} = 4\pi\rho r^2 \quad \Rightarrow \quad \rho_0 \sim \frac{M}{R^3} \tag{10}$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \Rightarrow \quad \frac{P}{R} \sim \frac{GM^2}{R^5} \tag{11}$$

$$\frac{dL}{dr} = 4\pi\rho r^2 \varepsilon, \;\; \varepsilon = \varepsilon_0 X^2 \rho T^4 \quad \Rightarrow \quad \frac{L}{R} \sim \frac{M^2 T^4 \varepsilon_0}{R^4} \tag{12}$$

$$\frac{dT}{dr} = -\frac{\kappa\rho}{16\pi r^2 acT^3} L, \;\; \kappa = \kappa_0 (X+1) Z \rho T^{-3.5} \quad \Rightarrow \quad \frac{T}{R} \sim \frac{\kappa_0 M^2 L}{R^8 T^{6.5}} \tag{13}$$

From equations (12) and (13) we find:

$$T \sim (\varepsilon_0 \kappa_0)^{2/7} \frac{M^{8/7}}{R^{20/7}}. \tag{14}$$

Then, from Eqs 9 and 14:

$$T \sim \frac{GM\mu_0}{R\Re} \quad (9)$$

$$T \sim (\varepsilon_0 \kappa_0)^{2/7} \frac{M^{8/7}}{R^{20/7}} \quad (14)$$

we find the relationship between the mass and radius of a star:

$$R \sim \frac{(\varepsilon_0 \kappa_0)^{2/13}}{(G\mu_0)^{7/13}} M^{1/13}. \quad (15)$$

Similarly, we get the temperature-mass relation:

$$T \sim M^{12/13},$$

and the luminosity-mass relation:

$$L \sim M^{71/13} \sim M^{5.46}.$$

The relations between the global properties and the element abundances can be obtained if we use power-law approximations for μ_0 , ε_0 , and κ_0 :

$\mu_0 \sim X^{\gamma}$,

where $\gamma = \dfrac{d \log \mu}{d \log X} \approx -\dfrac{5X}{5X+3} \simeq -0.54$

for $X = 0.7$.

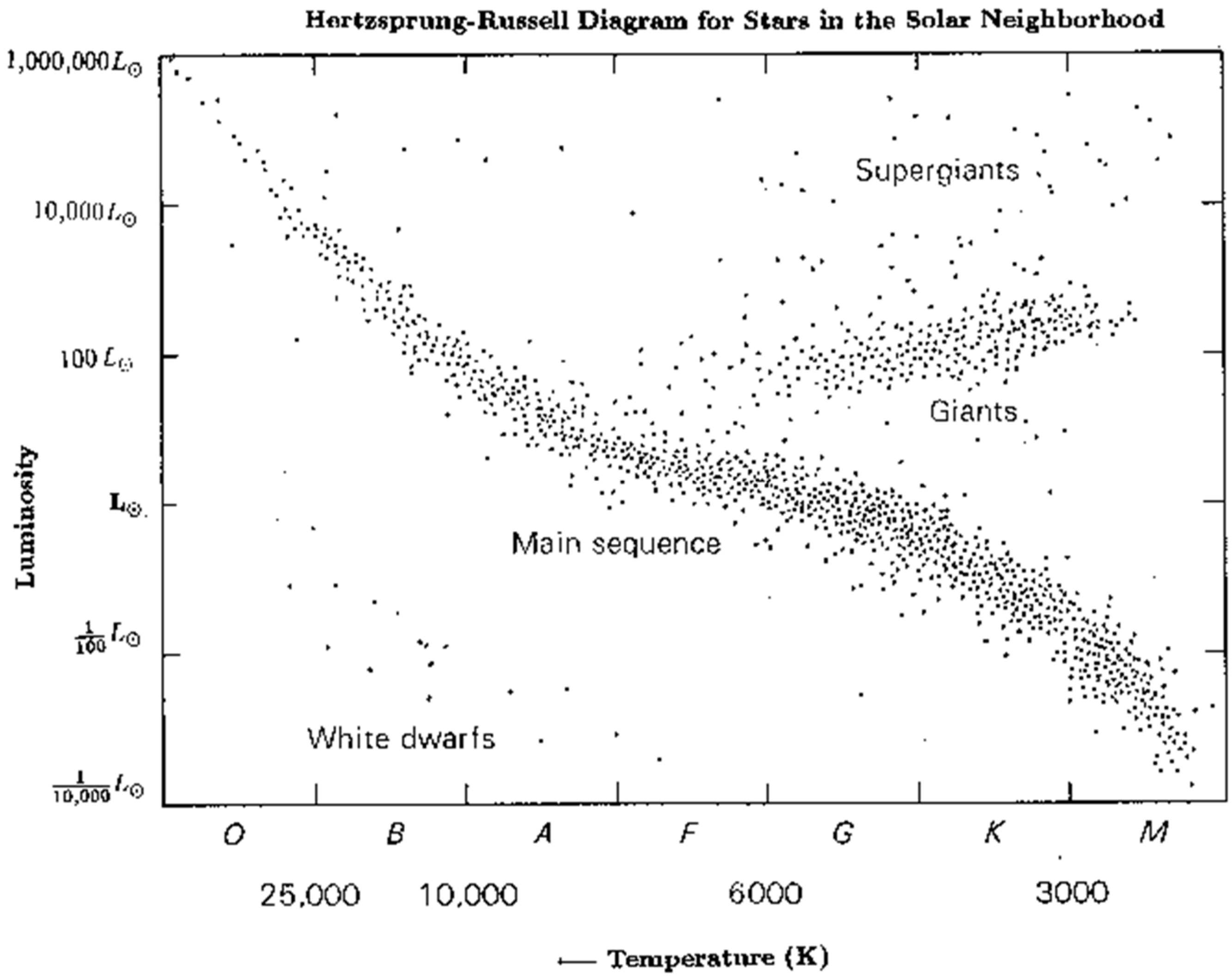
Then, $\kappa_0 \sim (X+1) \sim X^{\beta}$, where $\beta \sim \dfrac{1}{X+1} \simeq 0.59$; $\varepsilon_0 \sim X^2$.

Using these relations and Eq.(12-13) one can determine how the luminosity of the star depends on the hydrogen abundance and mass:

$$L \sim X^{-4.78} M^{5.46}.$$

It shows that the luminosity increases when the abundance of hydrogen decreases.

Exercise: Using the relation between the luminosity and the effective temperature: $L = 4\pi R^2 \sigma T_{\rm eff}^4$ estimate the slope of the Main sequence on the HR diagram, $-\dfrac{d\log L}{d\log T_{\rm eff}}$.



Evolution on the Main Sequence

The change of the hydrogen abundance can be described by the energy-balance equation which states that the luminosity of the Sun, L, is equal the energy release per unit mass, E, times the decrease rate of the hydrogen mass, $-M\frac{dX}{dt}$:

$$L = -EM\frac{dX}{dt}, \qquad (16)$$

where $E \approx 0.007c^2$ (the resulting helium has 0.7% less mass than the original hydrogen).

Lifetime on the Main Sequence

Equation for the hydrogen abundance:

$$L = -EM\,\frac{dX}{dt}, \qquad (16)$$

Luminosity-abundance scaling law: $L \sim X^{-4.78} M^{5.46}$.

Approximate the relationship between the luminosity and hydrogen abundance as:

$$L = L_0 \left(\frac{X}{X_0}\right)^{\alpha}, \qquad (17)$$

where $\alpha \approx -4.78$.

$$\frac{dX}{dt} = -\frac{L_0}{EM}\left(\frac{X}{X_0}\right)^{\alpha}, \qquad y = \frac{X}{X_0}, \quad a = \frac{L_0}{EMX_0}$$

$$\frac{dy}{dt} = -ay^{\alpha}, \quad \Rightarrow \quad \frac{dy}{y^{\alpha}} = -a \cdot dt \quad \Rightarrow \quad y^{(1-\alpha)} = a(1-\alpha)\left(\tau_n - t\right)$$

τ_n is the intergration constant

$$\left(\frac{X}{X_0}\right)^{(1-\alpha)} = a(1-\alpha)\tau_n\left(1 - \frac{t}{\tau_n}\right), \quad a(1-\alpha)\tau_n = 1 \text{ if } X = X_0 \text{ at } t = 0$$

$$\tau_n = \left(\frac{EMX_0}{(1-\alpha)L_0}\right)$$

Lifetime on the Main Sequence

Finally, the solution is:

$$X = X_0\left(1-\frac{t}{\tau_n}\right)^{\frac{1}{1-\alpha}},$$

$$L = L_0\left(1-\frac{t}{\tau_n}\right)^{\frac{\alpha}{1-\alpha}}, \quad \text{where} \quad \tau_n = \frac{MEX_0}{L_0(1-\alpha)}. \ \alpha \approx -4.78.$$

Then,

$$L = \frac{L_0}{(1-t/\tau_n)^{0.8}}. \qquad (19)$$

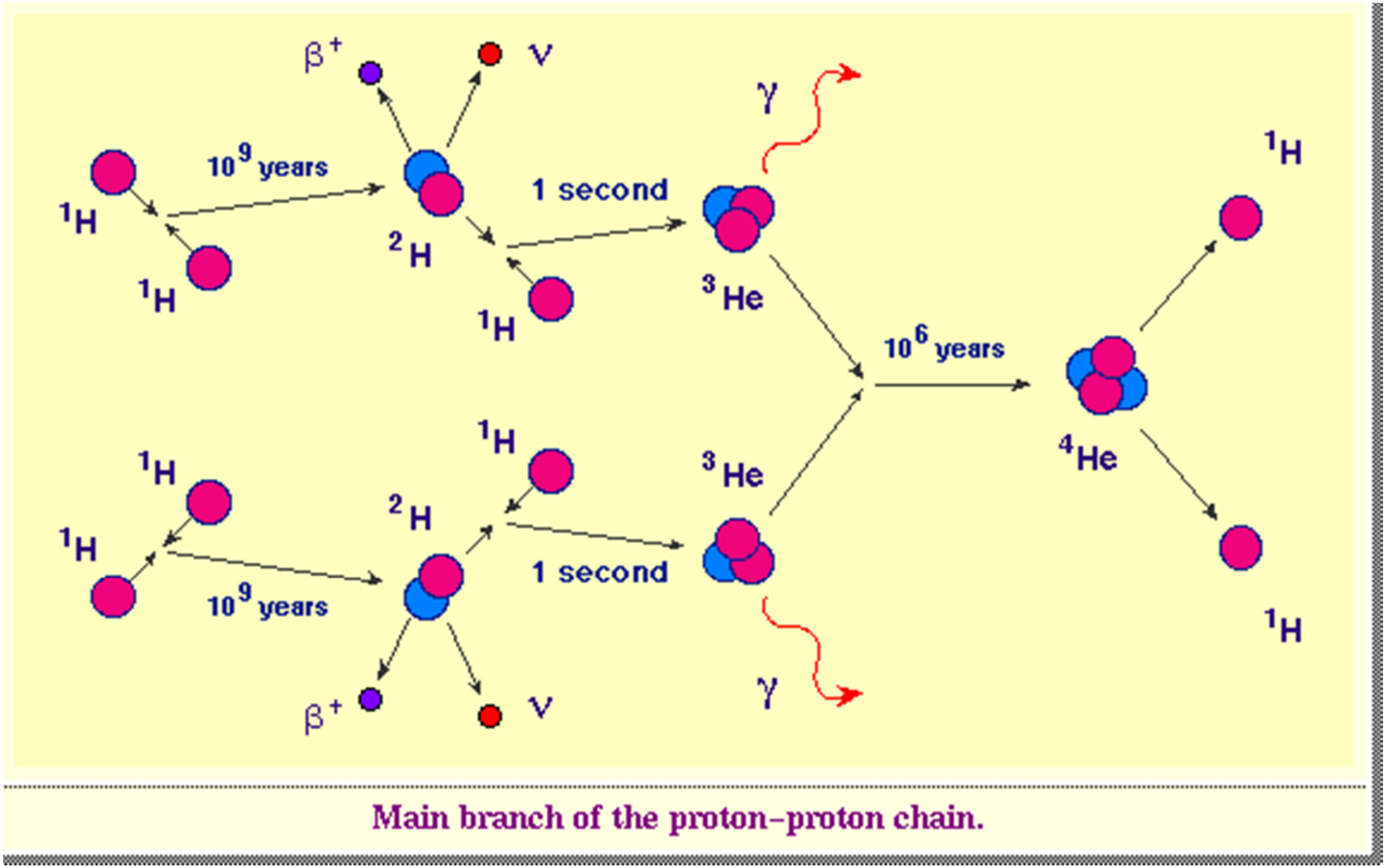
A finite solution exists only for $t < \tau_n$.

$$\tau_n \approx 0.17\frac{MEX_0}{L_0} \approx 0.17\frac{0.007c^2M}{L} \simeq 5.4\times10^{17}\,\text{s} \approx 1.7\times10^{10}\,\text{years}$$

is a characteristic time of the Sun's evolution on the Main Sequence ('nuclear time').

Nuclear Energy Sources: p-p chains

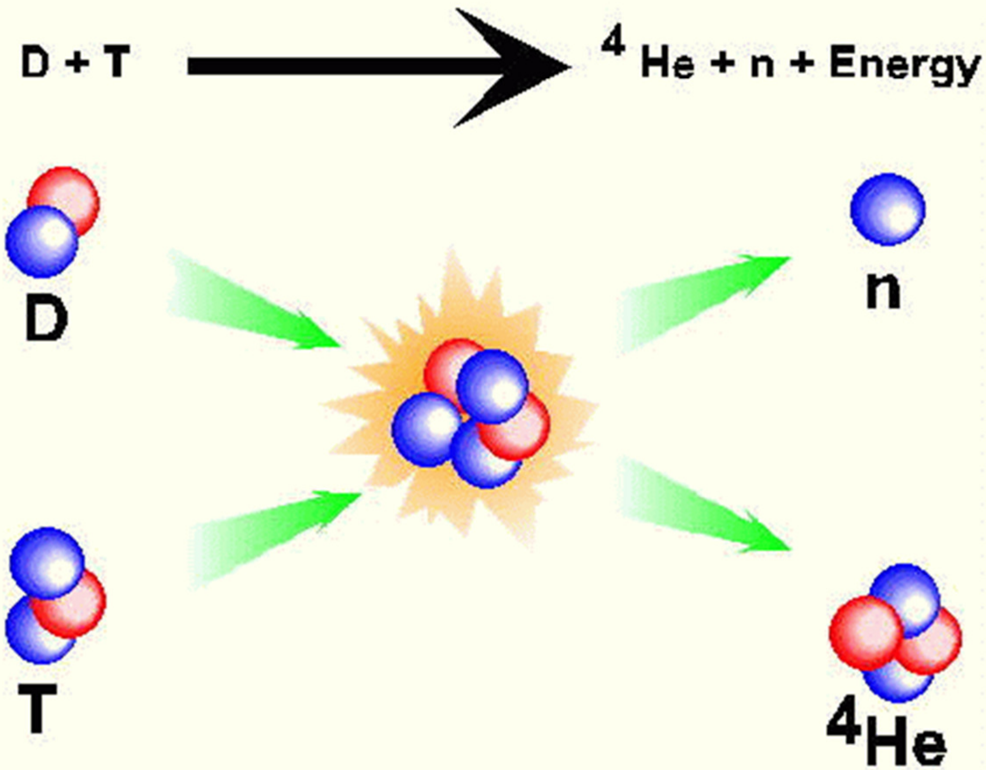
The main source of the solar energy is the proton-proton (pp) reaction which converts hydrogen to helium



Nuclear Fusion

Nuclear fusion is the process by which light elements combine to form heavier elements, giving off energy. The most common example of this is the fusion which takes place in the stars. In the core of the sun, at temperatures of 10–15 million degrees Celsius, hydrogen is converted to helium providing enough energy to sustain life on earth.

For an example of fusion in action, take the most suitable reaction for production of energy in proposed fusion reactors. It occurs between the nuclei of the two heavy forms (isotopes) of hydrogen, namely, deuterium (D) and tritium (T).



Deuterium and tritium fuse to form helium and a single neutron, giving off energy.

The p-p chain

$p + p \mapsto {}^{2}H + e^{+} + \nu_e$

$p + e^{-} + p \mapsto {}^{2}H + \nu_e$

$^{2}H + p \mapsto {}^{3}He + \gamma$

85 %

$^{3}He + {}^{3}He \mapsto {}^{4}He + 2p$

15 %

$^{3}He + {}^{4}He \mapsto {}^{7}Be + \gamma$

0.02 %

$^{7}Be + p \mapsto {}^{8}B + \gamma$

$^{8}B \mapsto {}^{8}Be^{*} + e^{+} + \nu_e$

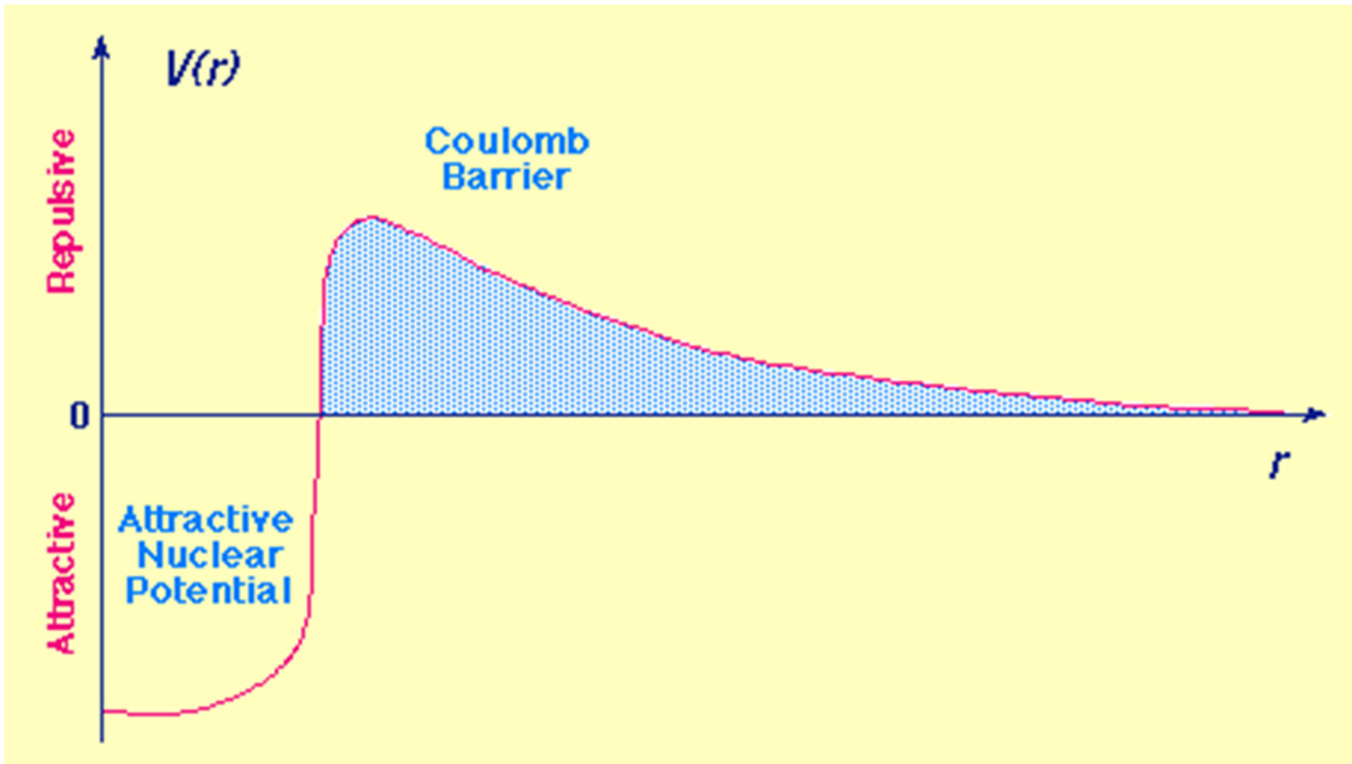
$^{8}Be^{*} \mapsto {}^{4}He + {}^{4}He$

$^{7}Be + e^{-} \mapsto {}^{7}Li + \nu_e$

$^{7}Li + p \mapsto {}^{4}He + {}^{4}He$

$^{3}He + p \mapsto {}^{4}He + e^{+} + \nu_e$

Nuclear potential



Nuclear Reactions

Nuclear reactions on the Sun are slow. They have low probabilities because of the high electrostatic barrier of the order of 1 MeV compared to the thermal energy of about 1 keV.

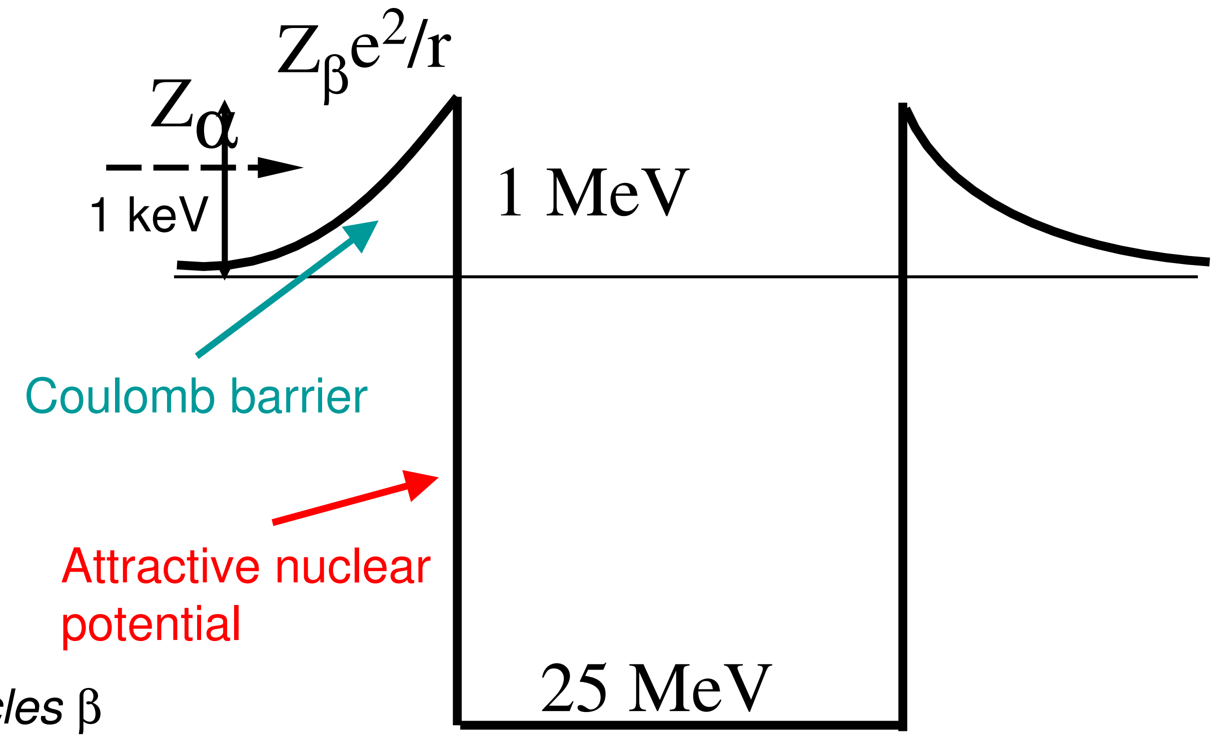
Particle flux: $n_\alpha v$

Reaction cross-section: σ

Reaction rate:

$$r_{\alpha\beta} = n_\alpha v \, \sigma \, n_\beta$$

density of particles β



Nuclear Barrier.

Consider the reaction rate between particles α and β per unit mass:

$$r_{\alpha\beta} \sim n_\alpha n_\beta < \sigma v >,$$

where σ is the reaction cross-section, and v is the relative velocity (the reaction rate is the product of the flux, $n_\alpha v$, of particles α, density n_β of the target particles and the reaction cross-section σ); $< ... >$ in an average over the particle velocity (or energy) distribution:

$$< \sigma v > = \int \sigma v \frac{dn}{n},$$

where dn/n is a fractional number of particles of in an energy interval $[E, E + dE]$.

For Maxwell-Boltzmann ideal gas:

$$\frac{dn}{n} = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} e^{-\frac{E}{kT}} E^{1/2} dE.$$

Then, $$r_{\alpha\beta} \propto \int e^{-\frac{E}{kT}} E \sigma dE$$

We estimate the reaction cross-section qualitatively assuming that the nuclear interaction happens at the scale of DeBroglie length,

$$\lambda_p = \frac{h}{p} = \frac{h}{\sqrt{2mE}} ,$$

where h is Plank's constant, p is the particle momentum, m is the mass, and E energy

(compare this with the photon wavelength $\lambda = c/\nu = (hc)/(h\nu) = (hc)/E = h/(E/c) = h/p$)

The cross-section is proportional to the squared DeBroglie length, λ_p^2, and the probability the Coulomb barrier penetration, $\exp(-E_e/E)$, where $E_e = \frac{Z_\alpha Z_\beta e^2}{\lambda_p}$ is the height of the barrier, Z_α and Z_β are the particle charges.

$$\lambda_p = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

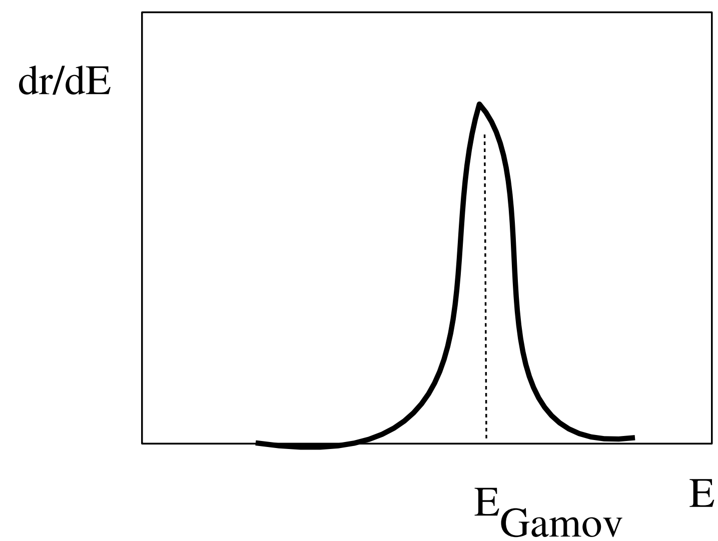
Then, $\sigma \sim \lambda_p^2 e^{-\frac{Z_\alpha Z_\beta e^2}{\lambda_p E}}$,

or $\sigma \sim \frac{h^2}{2mE} e^{-\frac{Z_\alpha Z_\beta e^2}{h}\sqrt{\frac{2m}{E}}} \propto \frac{1}{E} e^{-\frac{b}{E^{1/2}}}$,

where b is a constant.

Then, the reaction rate: $r \propto \int E e^{-\frac{E}{kT} - \frac{b}{E^{1/2}}} dE$.

The integrand has a sharp peak at $E = (bkT)^{3/2}$ - 'Gamov peak'.



Therefore,

$$r \propto \frac{1}{T^{2/3}} e^{-\frac{a}{T^{1/3}}},$$

where a is a constant.

The energy release in the nuclear reactions is:

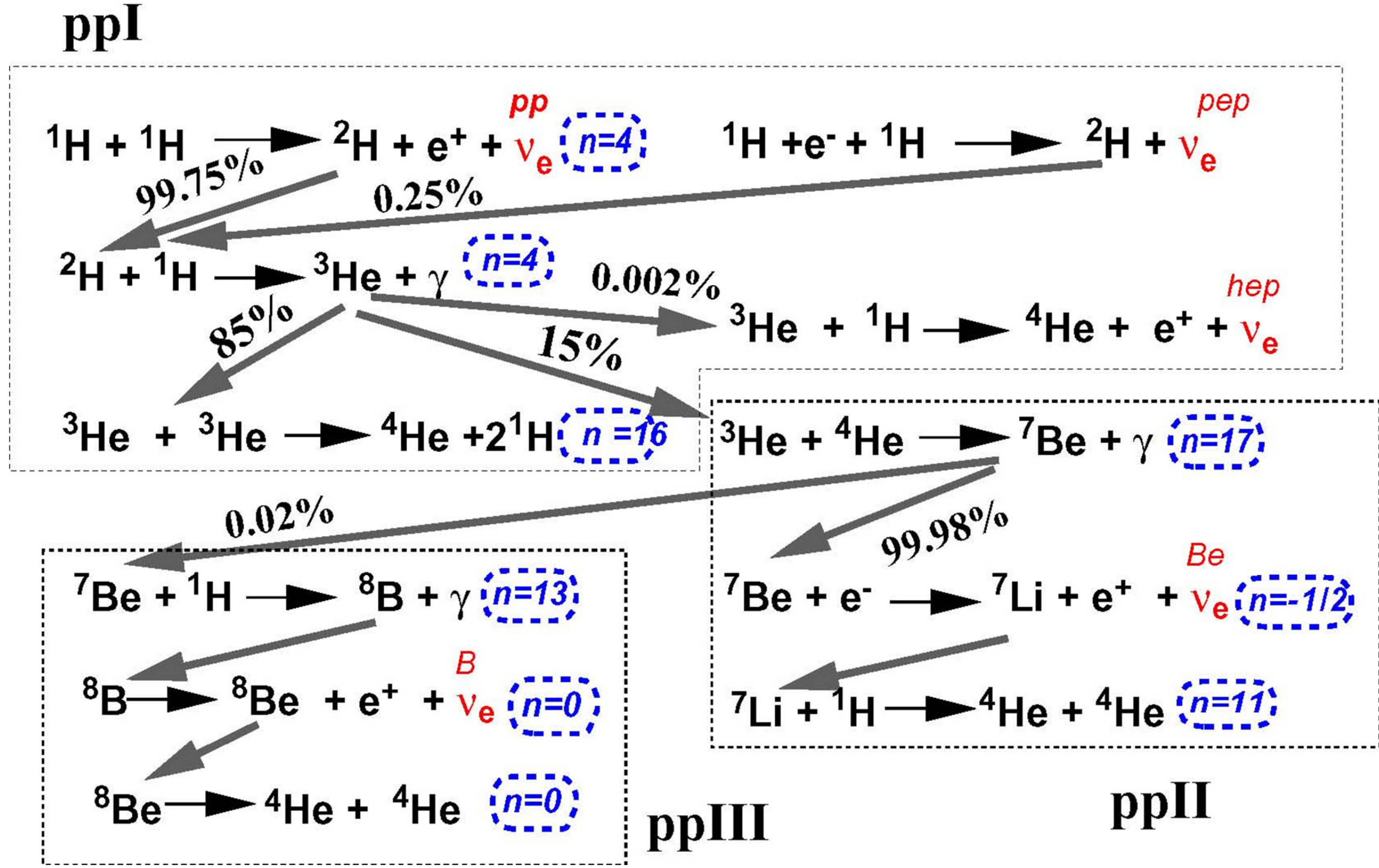
$$E \sim \Sigma Q_{\alpha\beta} r_{\alpha\beta} \propto \rho T^n,$$

where $n = \frac{a}{(kT)^{1/3}} - \frac{2}{3}$, or more precisely

$$n = \left(\frac{\pi^2 Z_\alpha^2 Z_\beta^2 e^4 m_{\alpha\beta}}{2h^2 kT} \right)^{1/3} - \frac{2}{3},$$

where $m_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}$.

The power index, n, is high for most reactions.



Nuclear reactions of the pp chain. The rates of the reactions depend on element abundances, density and temperature, e.g. for a reaction involving particles a and b, $\lambda_{ab} \propto \rho^2 X_a X_b T^n$. The power law index n is shown in the figure.

Abundances of the elements in the pp-chain are determined from the balance equation, e.g. for hydrogen abundance, $X \equiv X_1$:

$$\rho \frac{dX}{dt} = \rho^2 (-3\lambda_{11} X^2 + 2\lambda_{33} X_3^2 - \lambda_{34} X_3 X_4),$$

where X_3 is the abundance (mass fraction) of ^{3}He, $X_4 \equiv Y$ is the ^{4}He abundance

pp	$\rightarrow$	$^2H + e^+ + \nu_e$	
$^2H + p$	$\rightarrow$	$^3He + \gamma$	
$^3He + {}^3He$	$\rightarrow$	$^4He + 2p$	85%
$^3He + {}^4He$	$\rightarrow$	$^7Be + \gamma$	15%
$e^- + {}^7Be$	$\rightarrow$	$^7Li + \nu_e$	
$^7Li + p$	$\rightarrow$	$2\,{}^4He$	
$p + {}^7Be$	$\rightarrow$	$^8B + \gamma$	0.02%
^8B	$\rightarrow$	$^8Be^* + e^+ + \nu_e$	
$^8Be^*$	$\rightarrow$	$2\,{}^4He$	

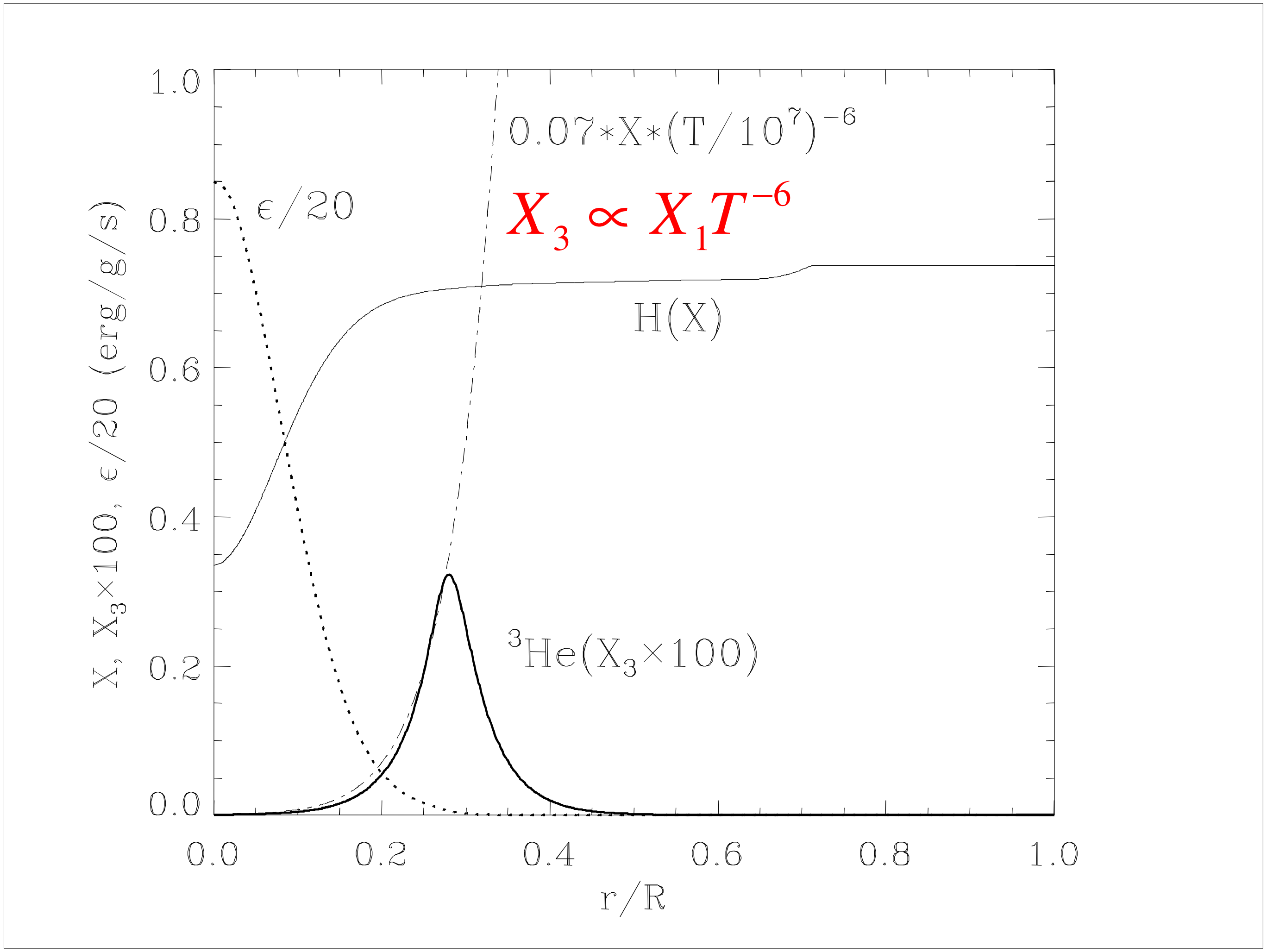
Estimates from the balance equations

Balancing the main terms in the nuclear reaction equations we obtain relations among various elements.

ppI:

$$\rho^2 X_1^2 T^4 \propto \rho^2 X_3^2 T^{16}$$

$$X_3 \propto X_1 T^{-6}$$



Abundances of hydrogen, X_1 and ^{3}He (X_3), and the the total energy-generation rate, ε. The ^{3}He production in the ppI chain increases with the radius following the simple power law, but drops to zero outside the energy-generating core because the balance equation is invalid there (the reaction is not in equilibrium).

pp	$\rightarrow$	$^2H + e^+ + \nu_e$	
$^2H + p$	$\rightarrow$	$^3He + \gamma$	
$^3He + {}^3He$	$\rightarrow$	$^4He + 2p$	85%
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$^7Li + p$	$\rightarrow$	$2\,{}^4He$	
$p + {}^7Be$	$\rightarrow$	$^8B + \gamma$	0.02%
^8B	$\rightarrow$	$^8Be^* + e^+ + \nu_e$	
$^8Be^*$	$\rightarrow$	$2\,{}^4He$	

ppII: $\rho^2 X_4 X_3 T^{17} \propto \rho X_7 n_e T^{-1/2}$

$$X_7 \propto \frac{\rho X_4 X_3 T^{11.5}}{n_e} \propto \frac{(1-X)X}{1+X} T^{11.5}$$

- abundance of ^{7}Be.

ppIII: $\rho^2 X X_7 T^{13} \propto \rho X_8$

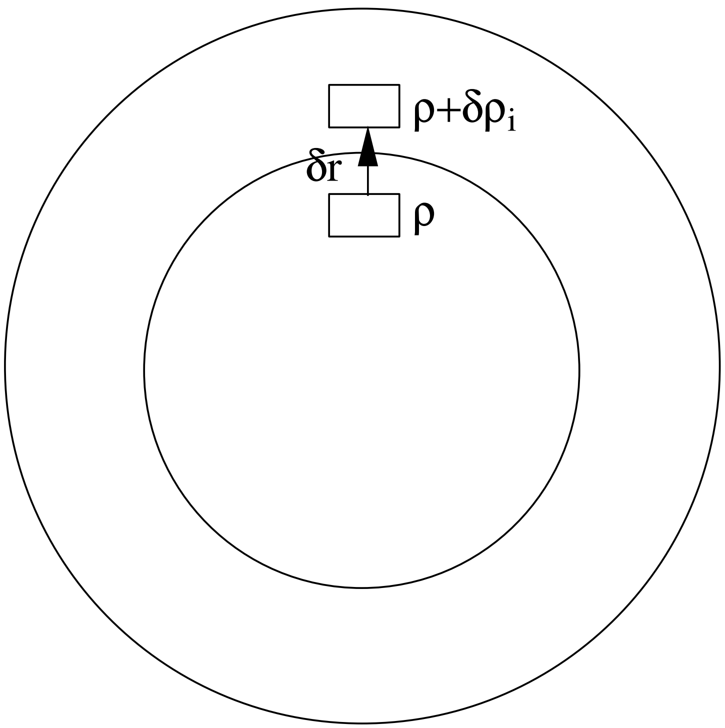
$$X_8 \propto \rho \frac{(1-X)X^2}{1+X} T^{24.5}$$

- abundance of ^{8}B which is the main source of high-energy neutrinos is very sensitive to temperature T the core.

Convective Instability

Consider a displacement, δr, of a small fluid element along the radius. If the density inside the displaced element, $\rho + \delta\rho_i$ is smaller the density of surrounding plasma, $\rho + \delta\rho$, then the element will continue moving up under the buoyancy force. Therefore, the condition of the convective instability is:

$$\Delta\rho \equiv \delta\rho_i - \delta\rho < 0.$$



Physical conditions inside the element obey the adiabatic law because the characteristic time for heat exchange is much longer than the dynamic time. Then,

$$\delta\rho_i = \left(\frac{d\rho}{dr}\right)_{\text{ad}} \delta r = \frac{\rho}{\gamma P}\left(\frac{dP}{dr}\right)\delta r,$$

where γ is the adiabatic exponent.

The density variation in the surrounding plasma is:

$$\delta\rho = \frac{d\rho}{dr}\delta r.$$

Finally, the instability condition is:

$$A^* \equiv \frac{1}{\gamma}\frac{d\log P}{d\log r} - \frac{d\log\rho}{d\log r} < 0.$$

Parameter A^* is called the Ledoux parameter of convective stability.

By using the equation of state, $\rho = \dfrac{P\mu}{\Re T}$, the condition of instability can be expressed in terms of temperature gradients:

$$\left(\frac{dT}{dr}\right)_{\rm ad} - \frac{dT}{dr} > 0, \ \text{or} \qquad \nabla_{\rm ad} - \nabla < 0,$$

where $\nabla_{\rm ad} \equiv \left(\dfrac{d\log T}{d\log P}\right)_{\rm ad}$ is the ‘adiabatic gradient’,

$\nabla \equiv \left(\dfrac{d\log T}{d\log P}\right)$ is the ‘local (ambient) gradient’.

For an ideal gas,

$$P = \Re\rho T / \mu, \quad P \propto \rho^{\gamma}$$

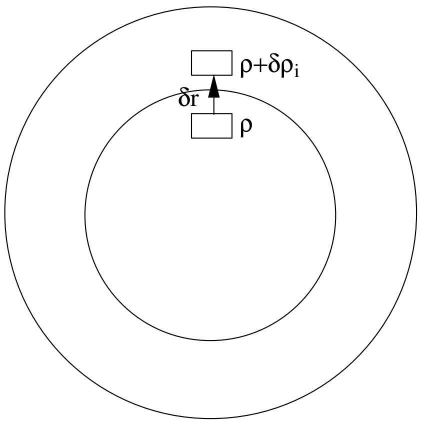
$$T \propto P^{\frac{\gamma-1}{\gamma}} \qquad .$$

$$\nabla_{\rm ad} \simeq \frac{\gamma-1}{\gamma} \approx 0.4$$

Convective Energy Transport. Mixing-Length Theory

Consider the momentum equation of a fluid element:

$\Delta\rho \equiv \delta\rho_i - \delta\rho$



$$\rho\frac{d^2\delta r}{dt^2} = -g\Delta\rho = -g\left[\left(\frac{d\rho}{dr}\right)_{\rm ad} - \frac{d\rho}{dr}\right]\delta r =$$

$$= -g\rho\left[\left(\frac{d\log\rho}{dr}\right)_{\rm ad} - \frac{d\log\rho}{dr}\right]\delta r =$$

$$= -g\rho\left[\left(\frac{d\log T}{d\log P}\right)_{\rm ad} - \frac{d\log T}{d\log P}\right]\frac{1}{H_P}\delta r,$$

where $\frac{1}{H_P} = -\frac{d\log P}{dr}$ is the pressure scale height; $P = \frac{\Re\rho T}{\mu}$

Finally, $$\frac{d^2\delta r}{dt^2} = -\frac{g}{H_P}(\nabla_{\rm ad} - \nabla)\delta r = -N^2\delta r, \qquad (22)$$

where $N^2 = \frac{g}{H_P}(\nabla_{\rm ad} - \nabla)$ is the Brunt-Väisälä frequency.

Another form of Brunt-Vaisala frequency:

$$N^2 = g\left(\frac{1}{\gamma P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right).$$

If $N^2 < 0$ then the medium is convectively unstable.

If $N^2 > 0$ then it is convectively stable.

In this case Eq.(22) has a solution $\delta r \propto \sin(Nt)$, that is the fluid elements oscillate with frequency N. These oscillations are called internal gravity waves - g-modes.

If we multiply Eq.(22)

$$\frac{d^2\delta r}{dt^2} = -\frac{g}{H_P}(\nabla_{\rm ad} - \nabla)\delta r = -N^2\delta r,$$

by $2\frac{d\delta r}{dt}$ and then integrate over t we get

$$\left(\frac{d\delta r}{dt}\right)^2 = \frac{g}{H_P}(\nabla - \nabla_{\rm ad})\delta r^2. \qquad (23)$$

The mixing-length theory assumes that the convective elements travel without destruction a distance l - 'mixing length'.

Then, the characteristic velocity, v, of these elements can be estimated from Eq.(23):

$$v^2 \sim \frac{g}{H_P}(\nabla - \nabla_{\rm ad})l^2.$$

Then, the convective energy flux is: $F_c = \rho v^2 \cdot v = \rho v^3 = \rho\left(\frac{g}{H_P}(\nabla - \nabla_{\rm ad})l^2\right)^{3/2}.$

The convective energy flux is: $F_c = \rho v^3 = \rho\left(\frac{g}{H_P}(\nabla - \nabla_{\rm ad})l^2\right)^{3/2}$.

In the convection zone, ∇ is slightly greater than $\nabla_{\rm ad}$. This is sufficient to carry the energy flux; the convective velocity is small in this regime, which is called 'efficient convection'.

However, near the solar surface where the density, ρ, is small much higher, near sonic velocity, is required to transport the solar energy. In this case, $\nabla >> \nabla_{\rm ad}$. This near surface zone is called a **'superadiabatic zone'**.

The mixing length is usually defined in terms of the pressure scale height:

$$l = \alpha H_P,$$

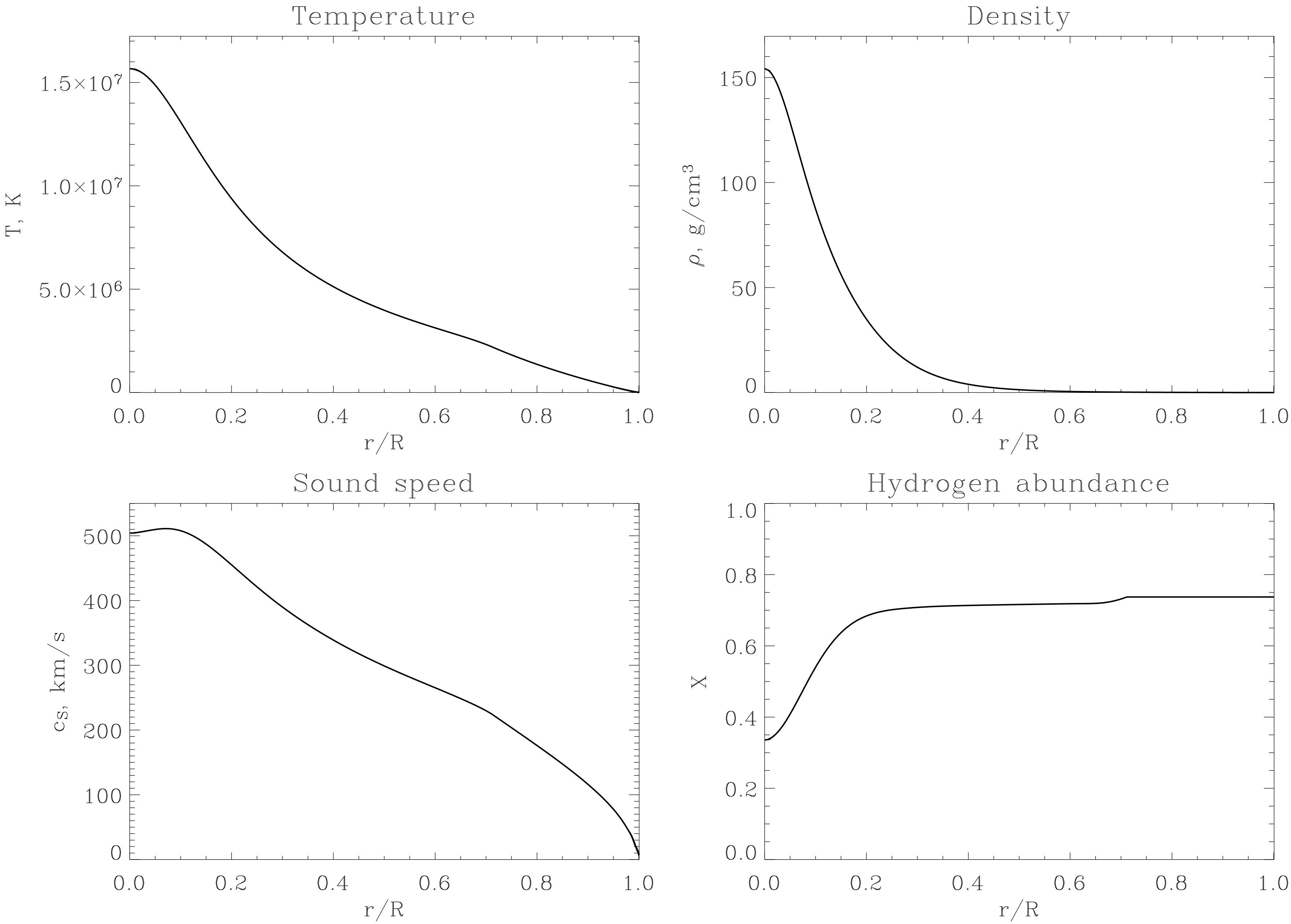
where α is called **'the mixing length parameter'**.

Comment on numerical modeling of the stellar structures.

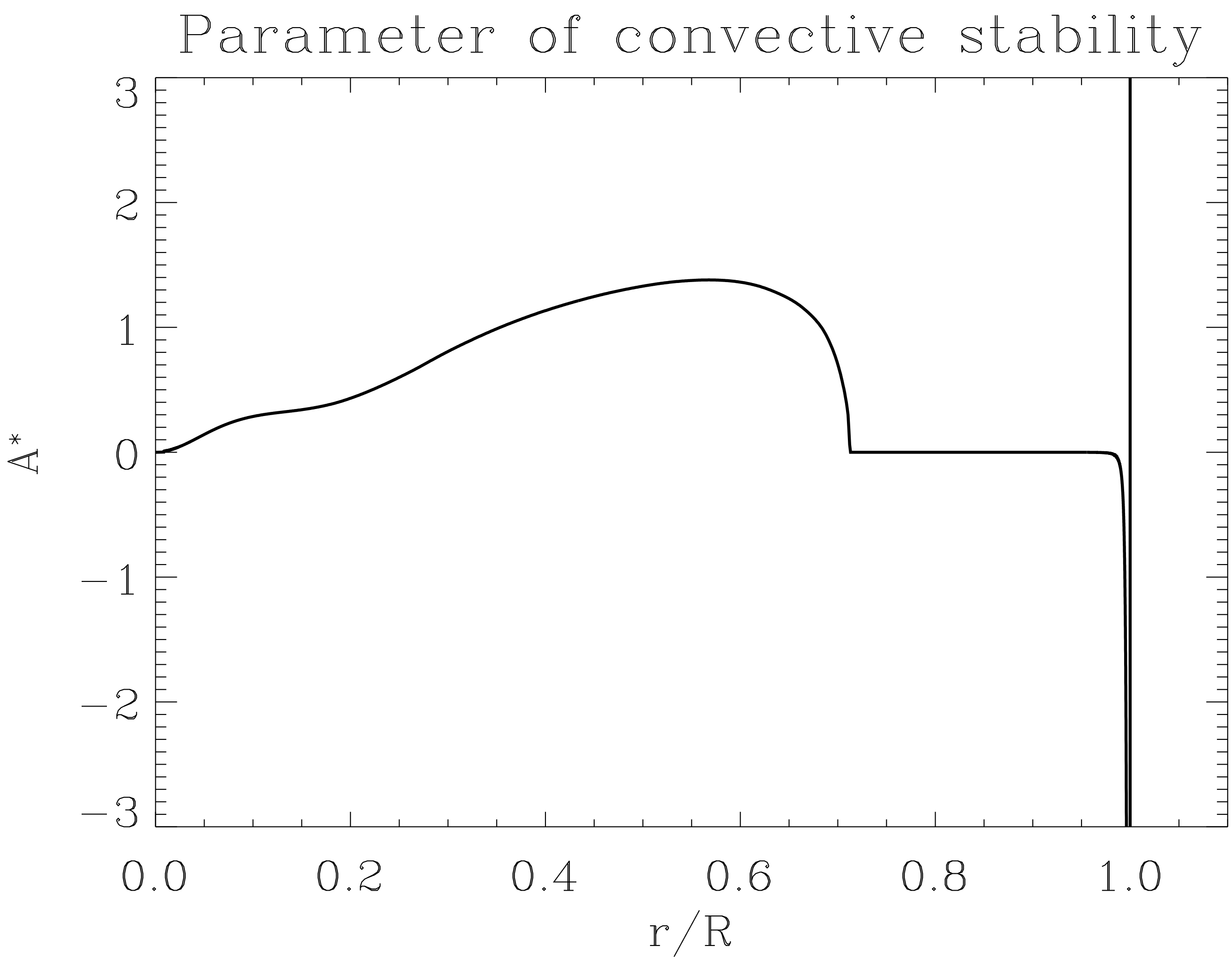
The stellar structure equations are solved for a star of mass M to match the observed (given) radius R and luminosity L.
The radius depends mostly on the mixing length parameter, α, and the luminosity depends mainly on the abundance of hydrogen, X, (or helium Y). Usually, α and Y are considered as free parameters to match R and L.

Online stellar modeling:
http://mesa-web.asu.edu/index.html

Standard solar model



Standard solar model



Gravitational Settling

It is assumed that initially, at the ZAMS, the Sun was chemically homogeneous because of mixing during the collapse phase. During the evolution on the main sequence it is well mixed in the convection zone. However, in the radiative zone there is element separation due to gravity. Elements heavier than hydrogen 'settle' under gravity.

Estimate the rate of the gravitational settling (diffusion). The settling speed can be estimated as: $v_D \sim g\tau_D$,

where τ_D is the mean collision time: $\tau_D \sim \frac{1}{n\sigma v_T}$, where n is the plasma density, σ is the collision cross-section, and v_T is the thermal velocity of ions.

Thus, $$v_D \sim \frac{g}{n\sigma v_T}.$$

For $g \sim 10^4$ cm/s^2, $n \sim 10^{23}$ cm^{-3}, $v_T \sim 10^6$ cm/s, and $\sigma \sim 10^{-16}$ cm^{-2}, we get $v_D \sim 10^{-8}$ cm/s.
The characteristic diffusion length during the previous solar evolution is: $L \sim v_D t_\odot \sim 1.5\times10^9$ cm.
This is significantly less than the radius of the radiative zone, $\sim 5\times10^{10}$ cm.

Nevertheless, taking into account the gravitation settling is important for an accurate modeling the solar structure. The abundance of helium in the convection zone has decreased due the gravitational settling from 0.28 to 0.25, by almost 10%.

Solar Neutrino Flux

The total neutrino flux can be estimated from the nuclear reaction rate:

$$\Phi_{\odot} = \frac{2L_{\odot}}{Q - 2E_{\nu}} \frac{1}{4\pi D^2} \approx 6.51\times 10^{10}\,\mathrm{cm}^{-2}\mathrm{s}^{-1},$$

where $D \approx 1.5\times 10^{13}$ cm is the distance from the Sun, $Q = 26.733$ MeV is the energy output of the reaction $4\,{}^{1}\mathrm{H} \to {}^{4}\mathrm{He}$; each reaction produces 2 p-p neutrinos, $E_{\nu} \approx 0.265$ MeV is the energy of neutrino in each of p-p chains.

The total neutrino flux consists mainly of p-p, ${}^{7}\mathrm{Be}$ and ${}^{8}\mathrm{B}$ neutrinos:

$$\Phi_{\odot} \approx \Phi_{\nu}(\mathrm{p-p}) + \Phi_{\nu}({}^{7}\mathrm{Be}) + \Phi_{\nu}({}^{8}\mathrm{B}).$$

Estimate the flux of ${}^{8}\mathrm{B}$ neutrinos by counting the relative rate of the ppIII chain:

$$\Phi_{\nu}({}^{8}\mathrm{B}) \sim 2\cdot 0.015\cdot 0.0002\Phi_{\odot} \simeq 6\times 10^{-5}\Phi_{\odot} \simeq 4\times 10^{6}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}.$$

The ${}^{8}\mathrm{B}$ neutrinos are observed through the reaction:

$$\nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}.$$

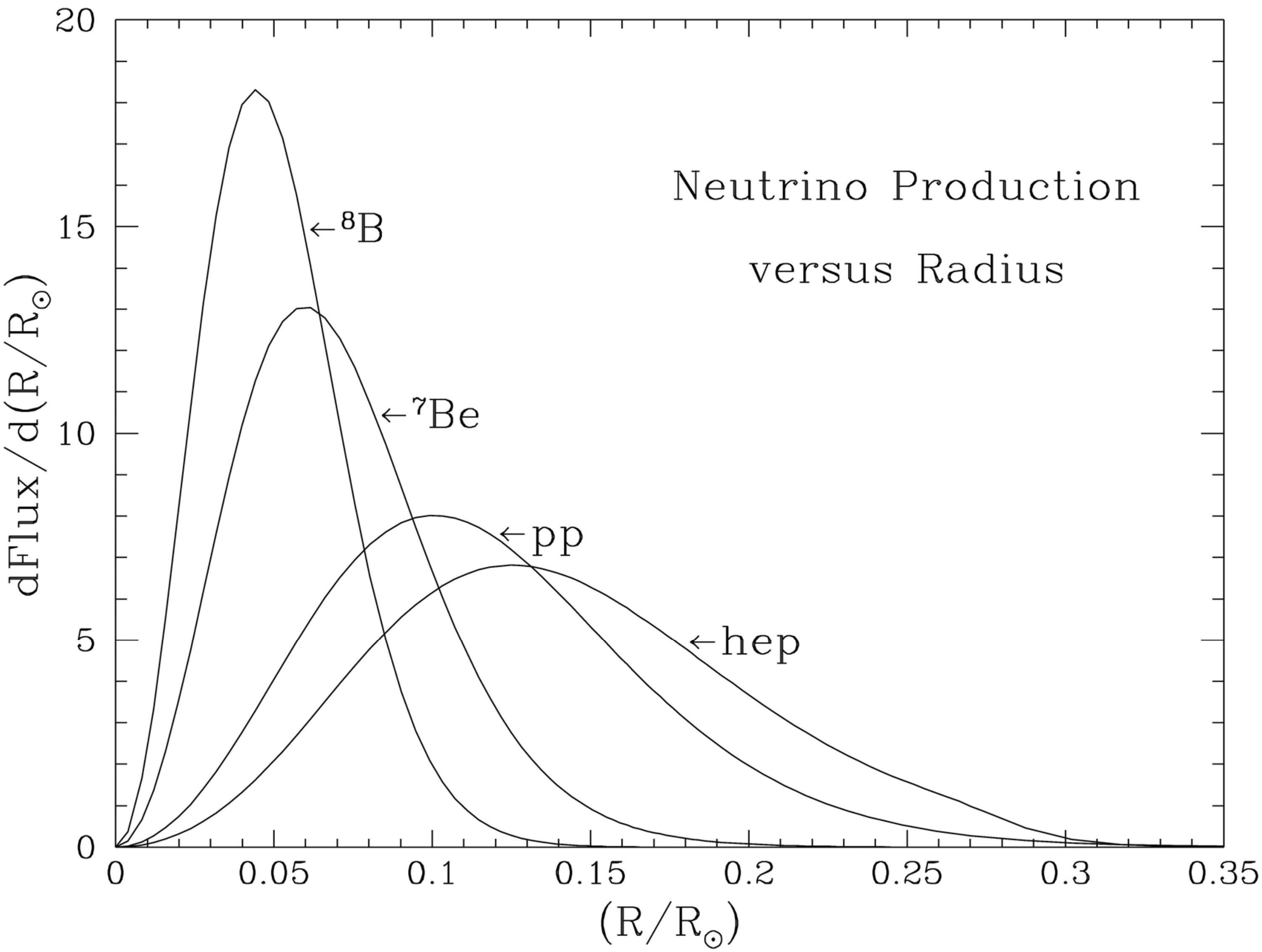
$$\Phi_{\mathrm{obs}} = <\sigma_{\mathrm{Ar}}\Phi_{\nu}> N \simeq 4.5\times 10^{-36} N\mathrm{s}^{-1},$$

where the cross-section of the reaction $\sigma_{\mathrm{Ar}} \approx 1.11\times 10^{-42}$ cm^2, N is the number of the target atoms (${}^{37}\mathrm{Cl}$).

For a reference number of the target atoms, $N = 10^{36}$:

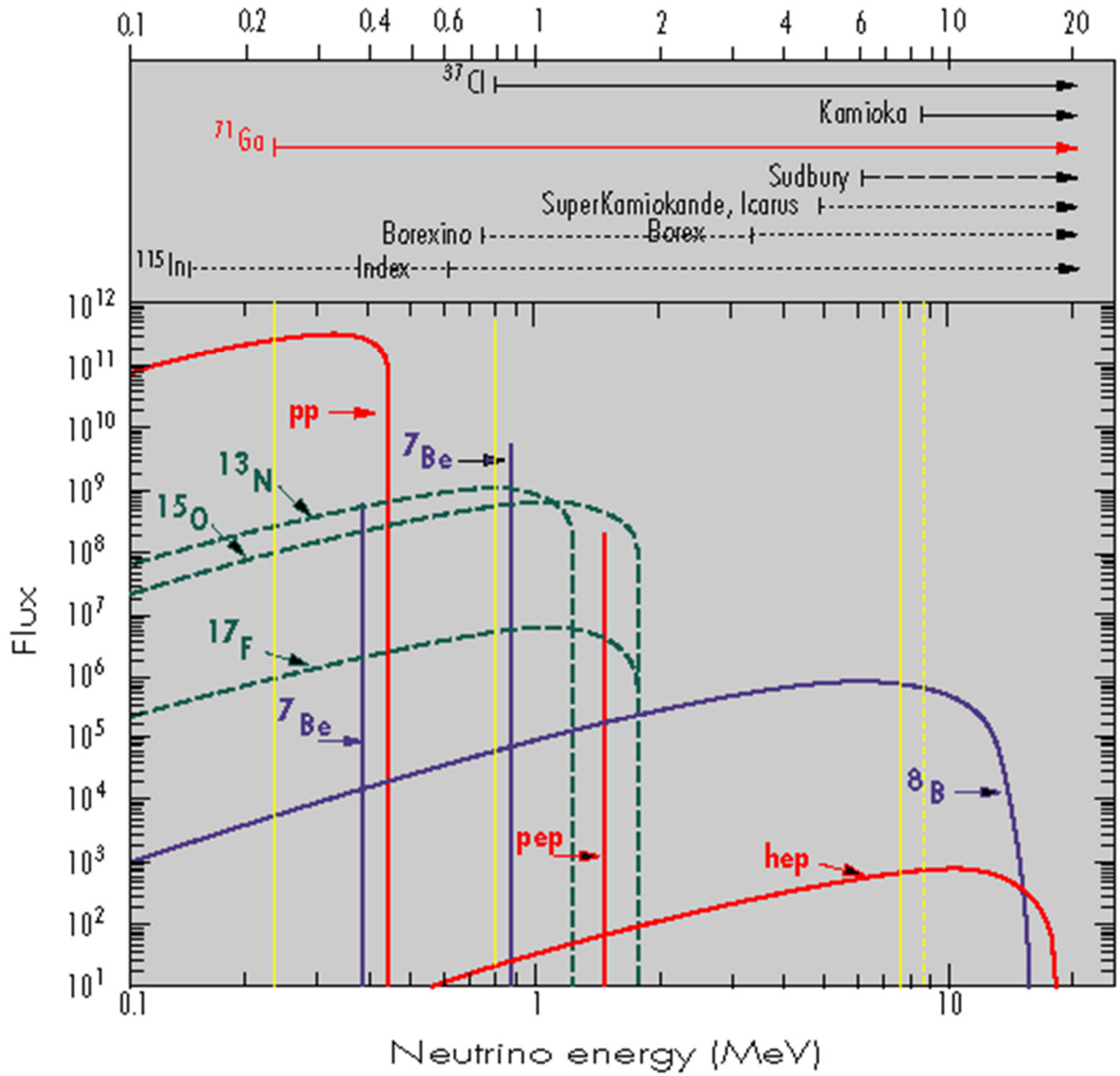
$$\Phi_{\mathrm{obs}} \approx 4.5\,\mathrm{SNU},$$

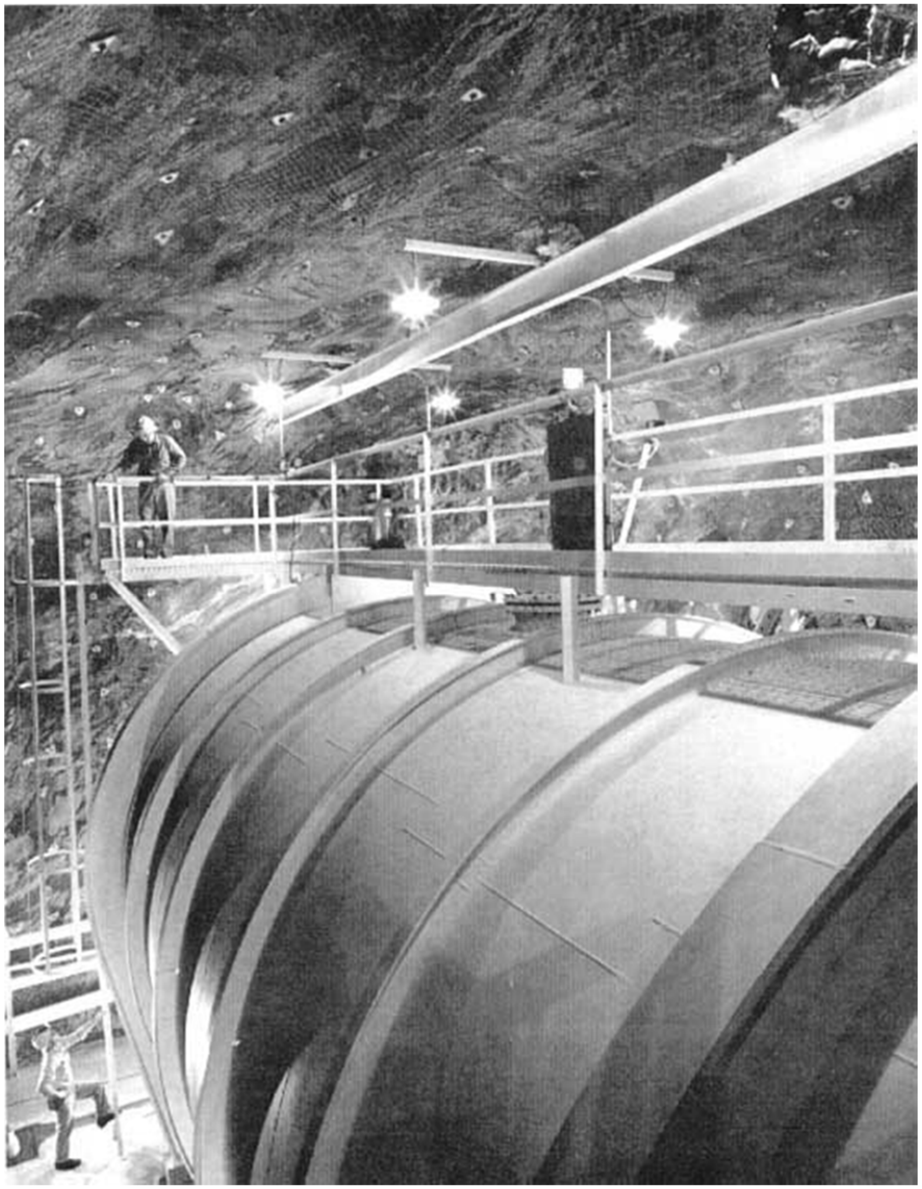
where **SNU is a 'solar neutrino unit' - the number of the reaction per sec per** 10^{36} **atoms.**



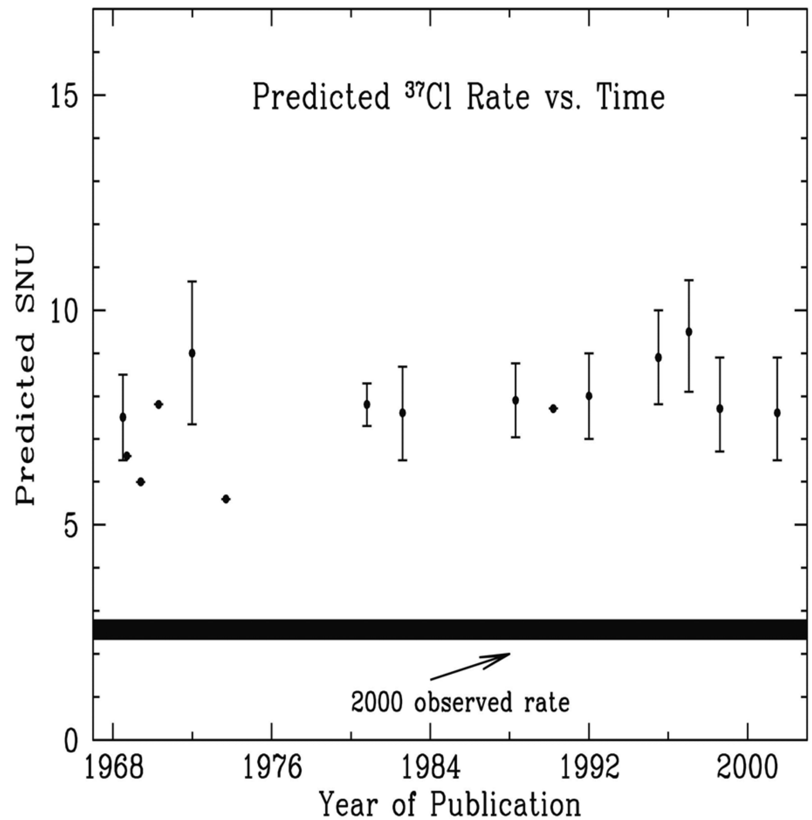
Neutrino Production
versus Radius
←^{8}B
←^{7}Be
←pp
←hep
dFlux/d(R/R$_\odot$)
(R/R$_\odot$)
0
5
10
15
20
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Sensitivity of various neutrino experiments





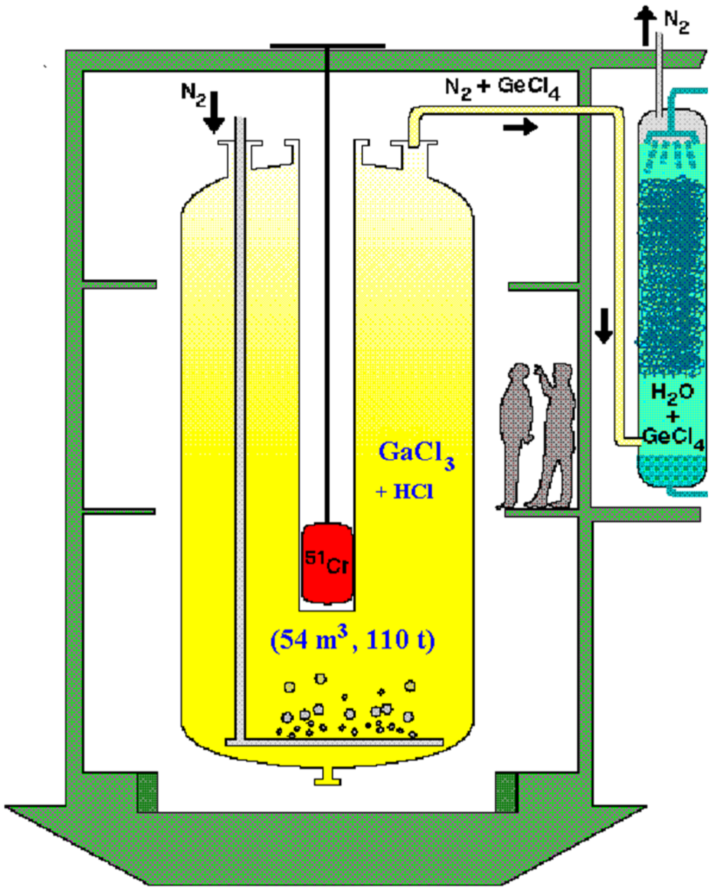
^{37}Cl detector

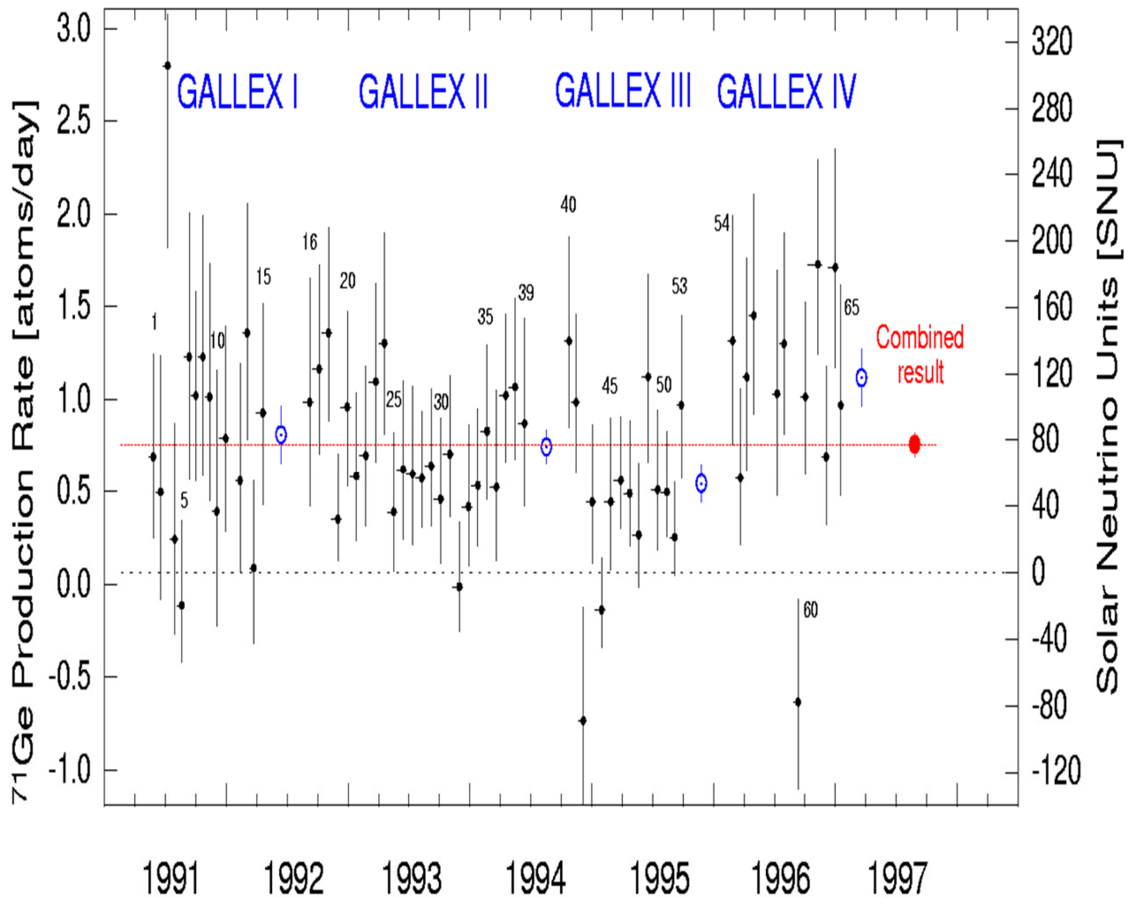


GALLEX experiment

Result: 77.5 8 SNU

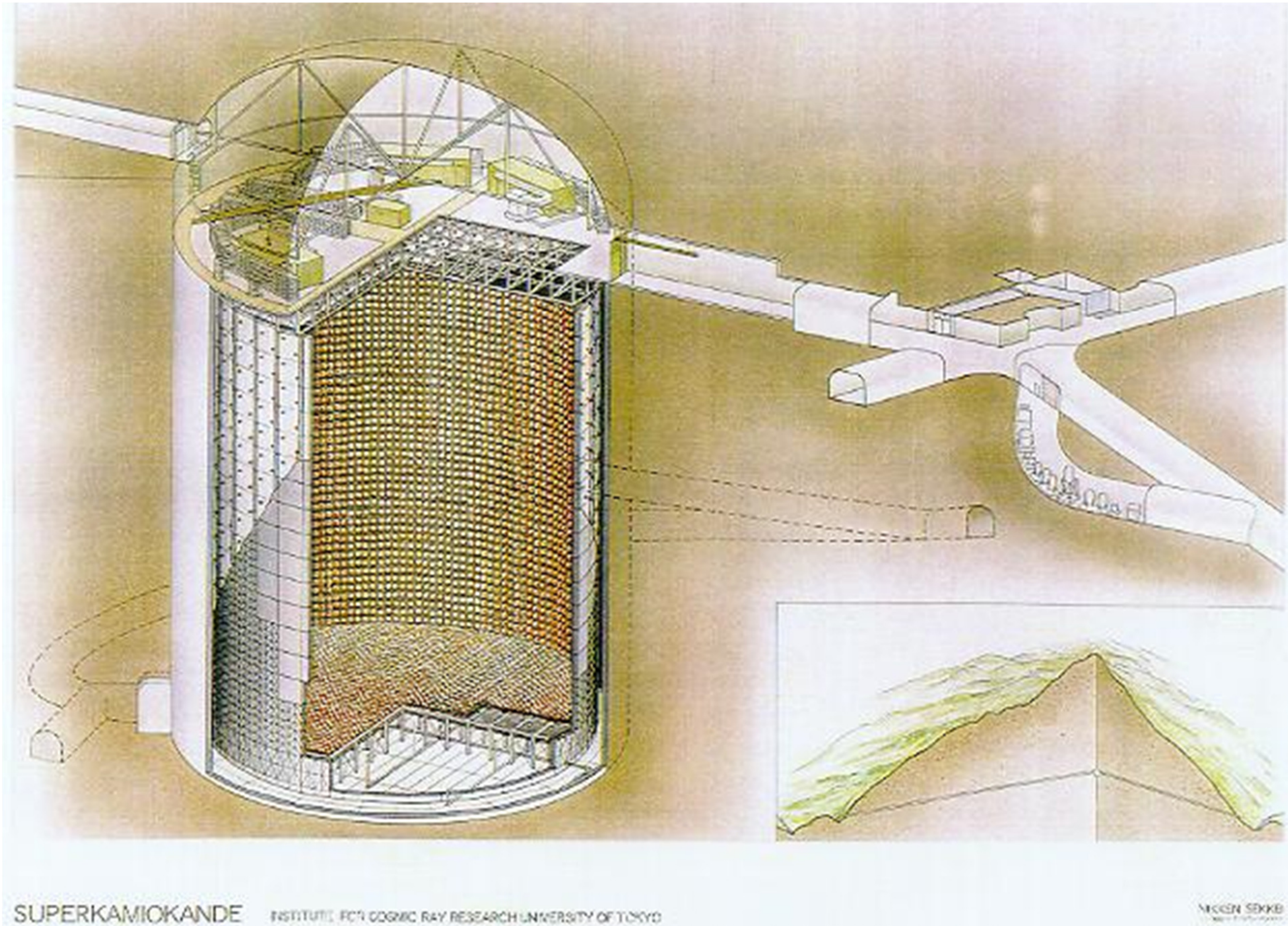
Standard Solar Model prediction: 129.6 SNU

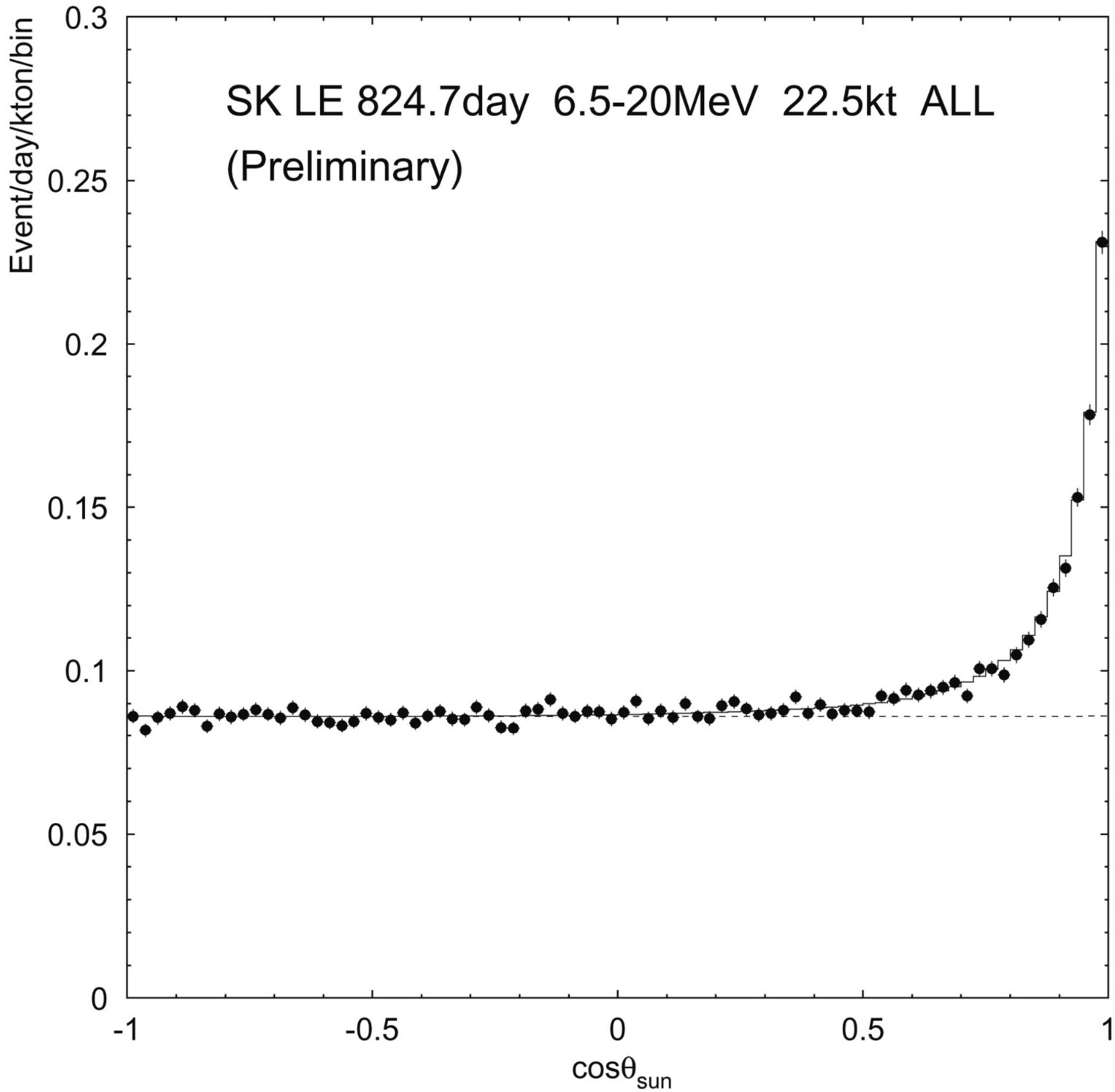




$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

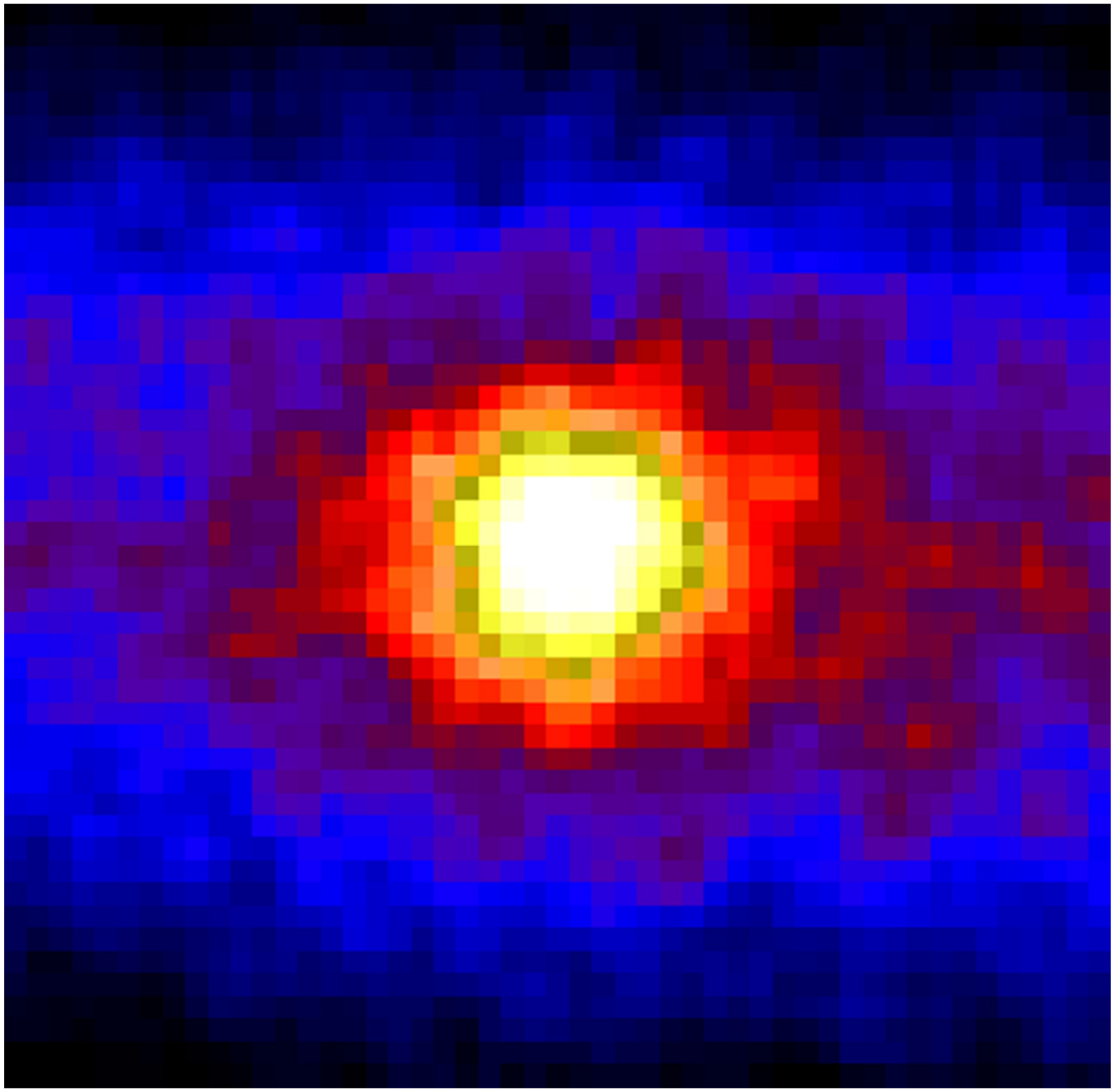
Superkamiokande (Japan)





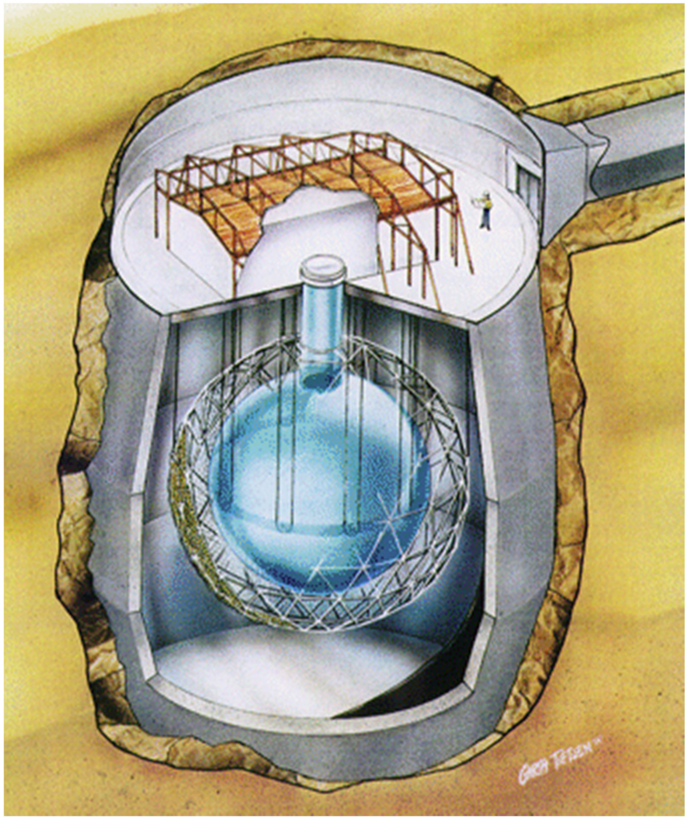
SK LE 824.7day 6.5-20MeV 22.5kt ALL
(Preliminary)
Event/day/kton/bin
0.3
0.25
0.2
0.15
0.1
0.05
0
-1
-0.5
0
0.5
1
cosθsun

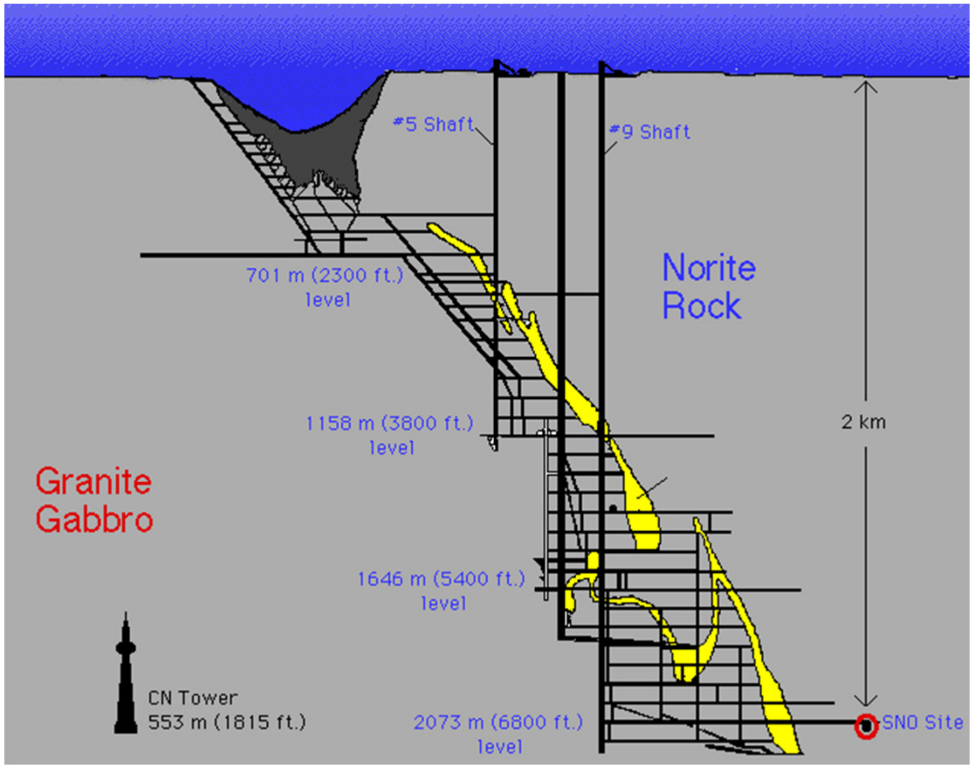
Neutrino image of the Sun

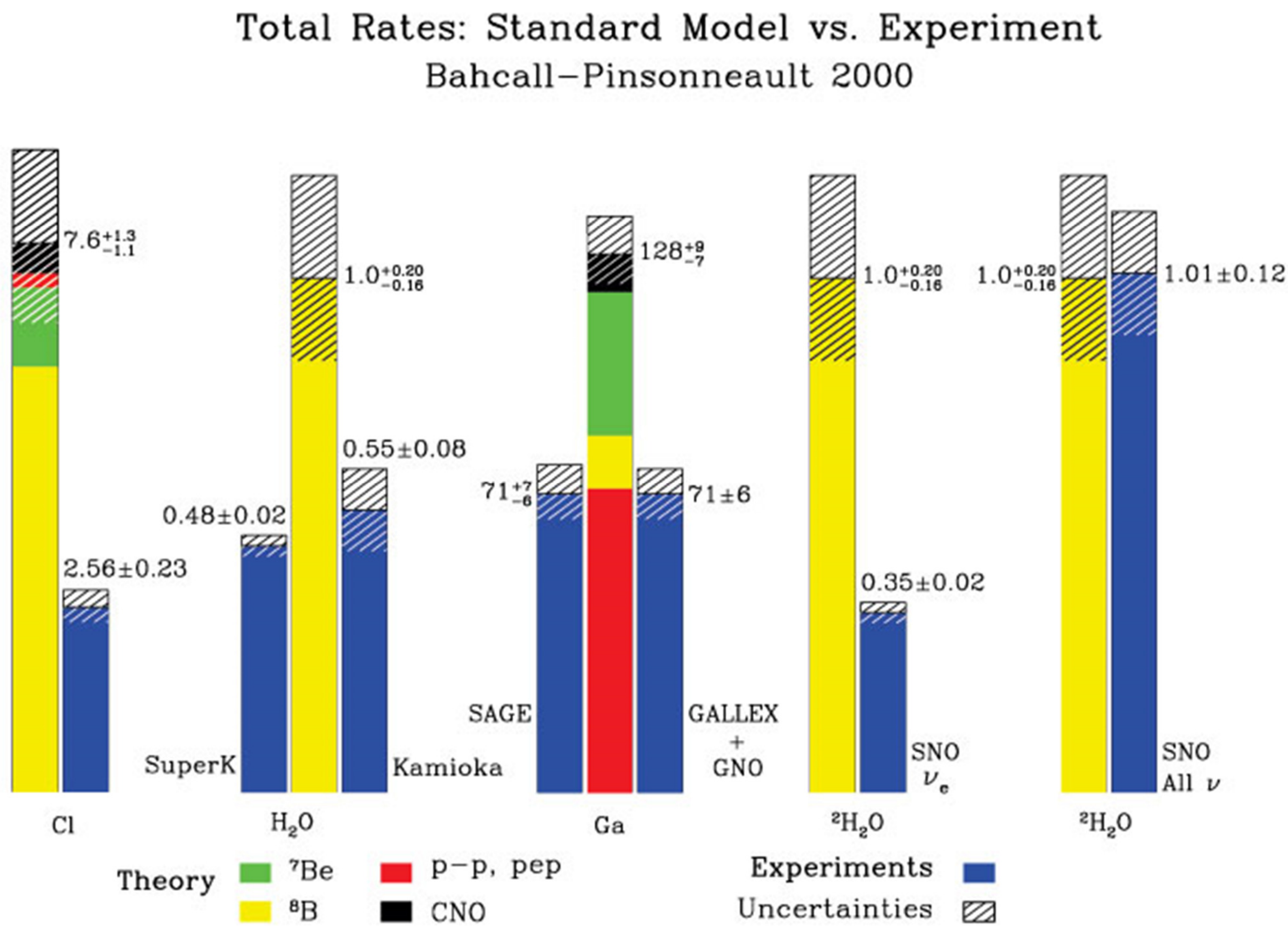


Sudbury Neutrino Observatory (SNO)

measured the total flux of electron, muon and tau neutrinos and confirmed the neutrino transition from one type to another on their way from the Sun to Earth







$v_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$. $v_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$

Neutrino Flavor Composition of ^8B Flux

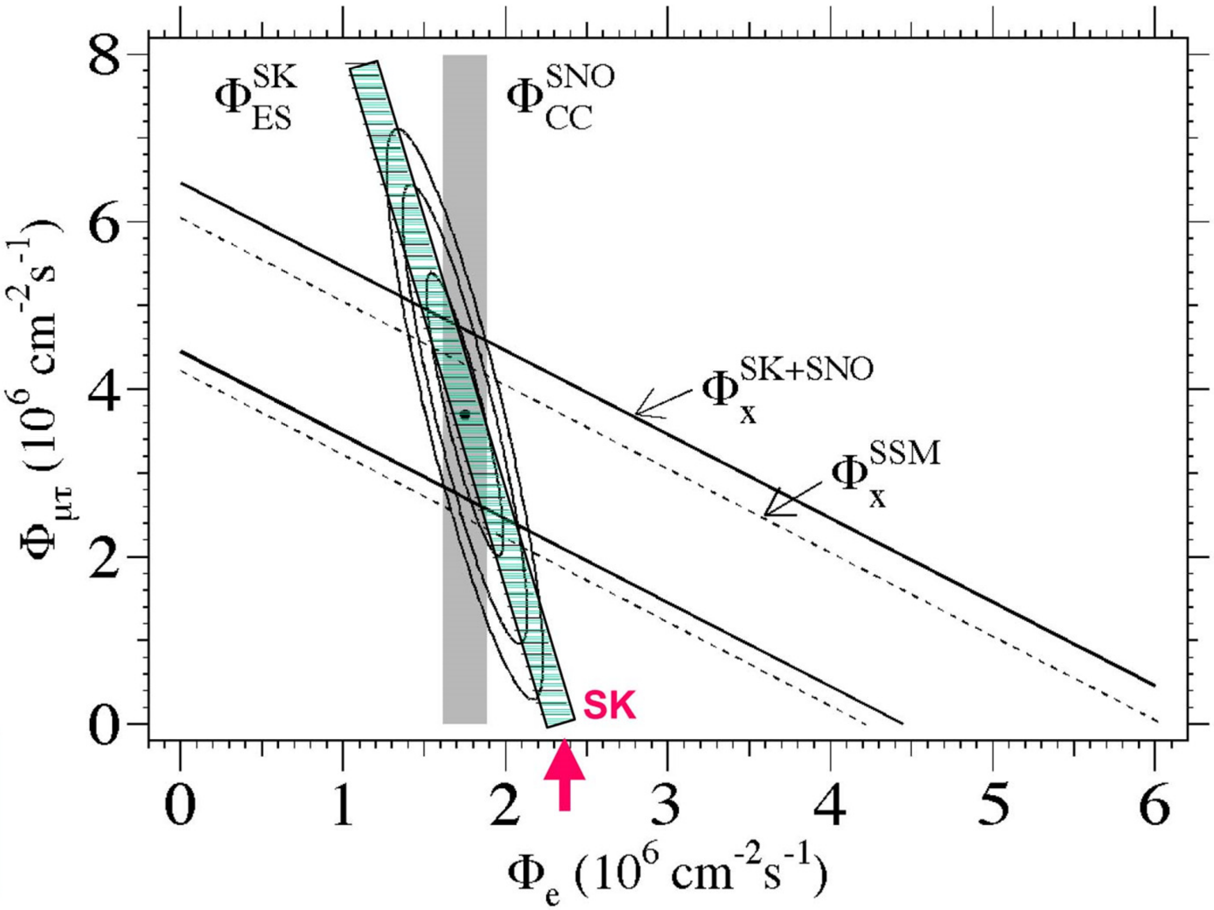
Fluxes

(10^6 cm^{-2} s^{-1})

ν_e:	1.75(15)
$\nu_{\mu\tau}$:	3.69(113)
ν_{total}:	5.44(99)
ν_{SSM}:	5.05

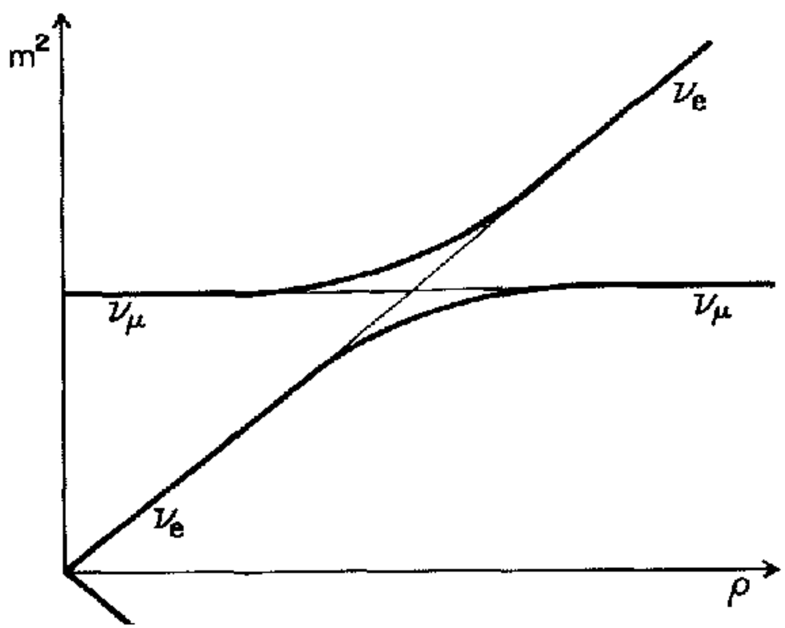
$\phi_{CC} = \phi_e$

$\phi_{ES} = \phi_e + 0.154\,\phi_{\mu,\tau}$



MSW (Mikheyev-Smirnov-Wolfenstein) effect

The MSW effect states that any coherent forward scattering of electron neutrinos in electronic matter results in a density-dependent effective mass. This means that electron neutrinos which travel through an inhomogeneous medium (like the density gradients in the sun or the earth) have some probability of changing their effective mass and hence their flavor.



The masses of two flavors of neutrino as a function of density.

The probability of transition between the states is high in the avoided crossing region (MSW-effect).

Mixing angle: $\bar{\nu}_e = \nu_e \sin\theta + \nu_\mu \cos\theta.$

The mixing angle should be small to explain the deficit of ν_e .