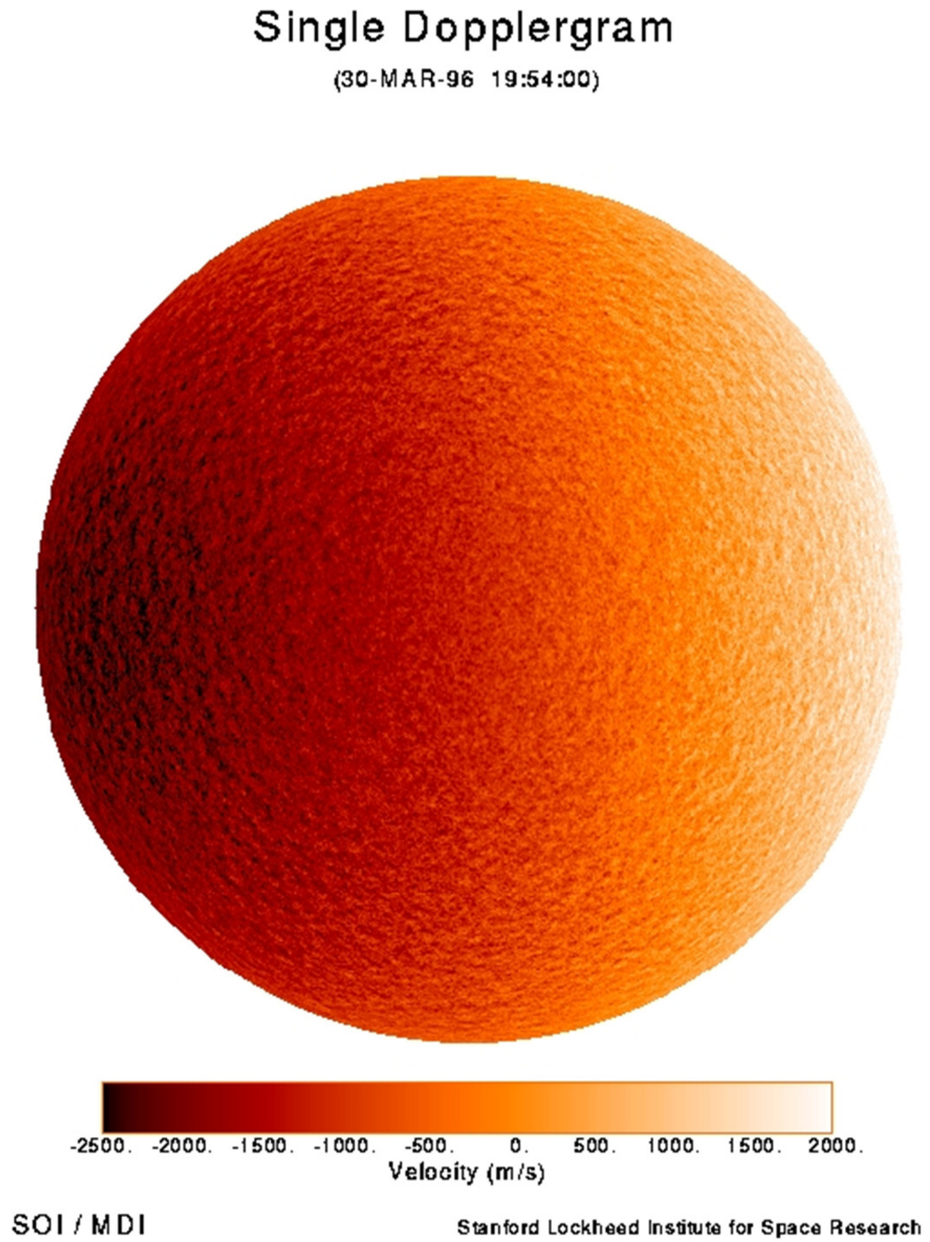
8. Solar oscillations

Outline

- Observations
- Theory of p-, g-, and r-modes.
- Excitation mechanisms.



The rotation speed of the solar surface is 2km/s.

Average Dopplergram Minus Polynomial Fit

45 images averaged (30-Mar-96 19:26 to 30-Mar-96 20:17)

SOI / MDI

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Single Dopplergram Minus 45 Images Average

(30-MAR-96 19:54:00)

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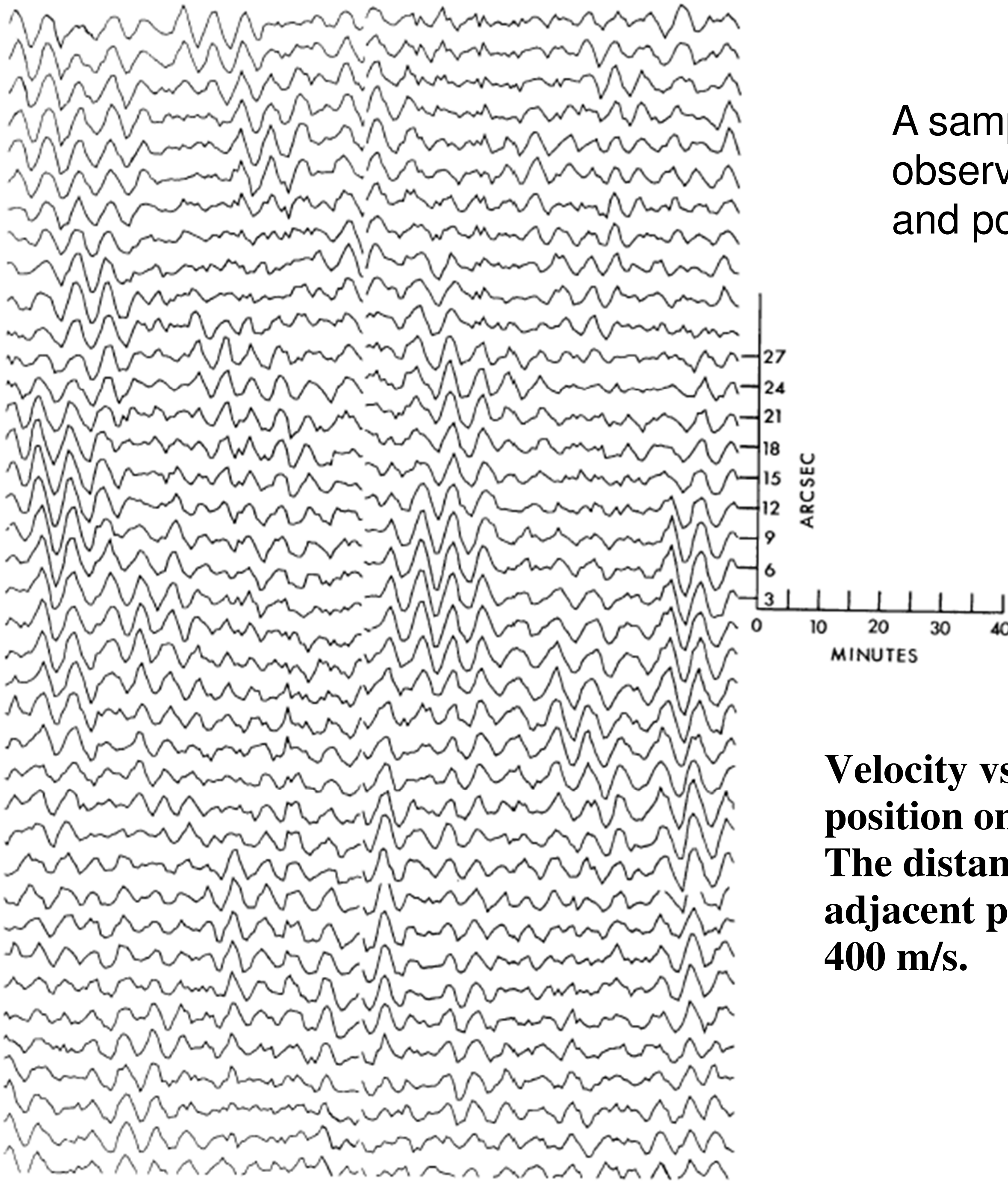
Observations of Solar Oscillations

Solar oscillations were discovered in 1960 by Leighton, Noyes and Simon by observing Doppler shift in spectroheliograms.

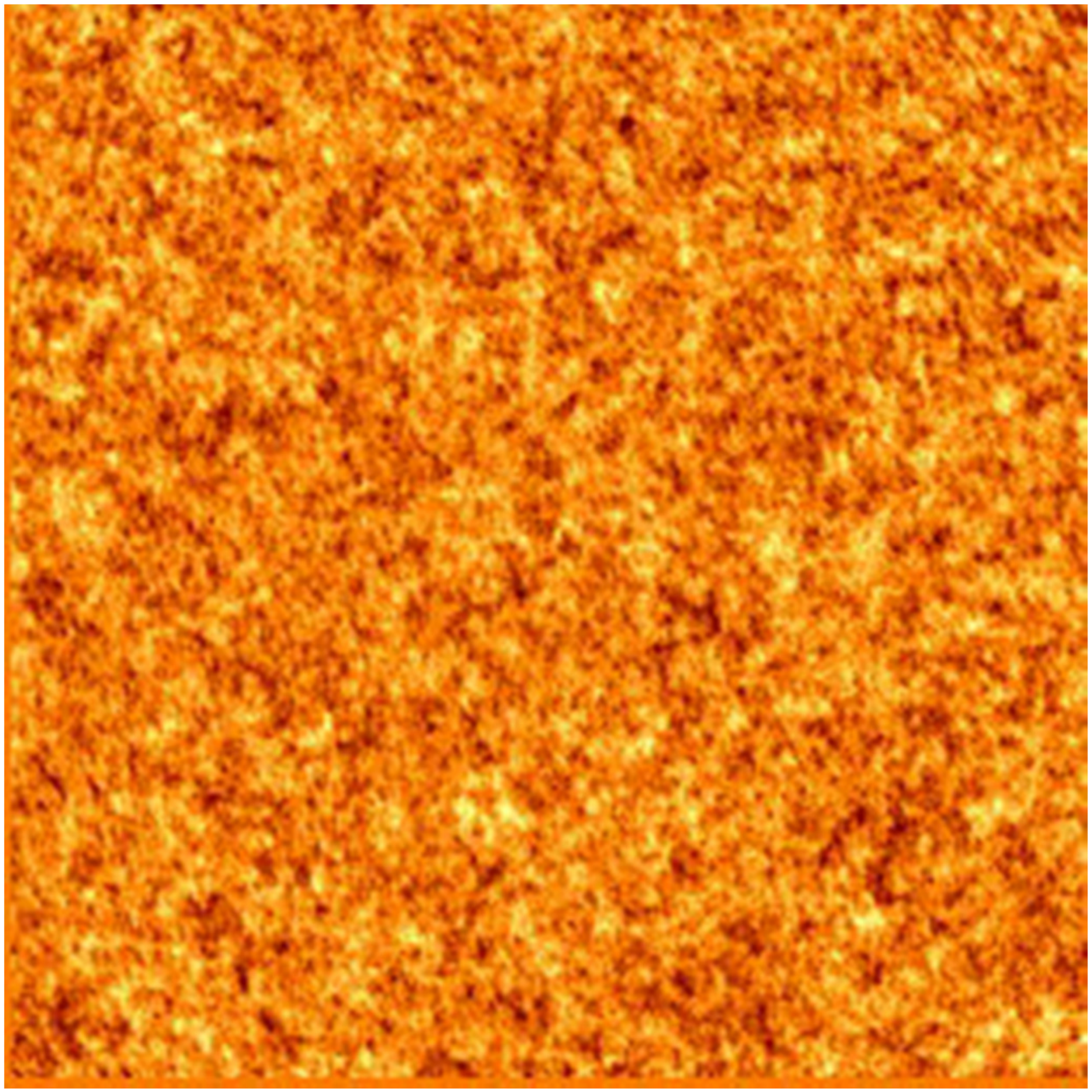
MDI single Dopplergram minus an average solar velocity image observed over 45 minutes reveals the surface motions associated with sound waves traveling through the Sun's interior. The small scale light and dark regions represent the up and down motions of the hot gas near the Sun's surface. The pattern falls off towards the limb because the acoustic waves are primarily radial.



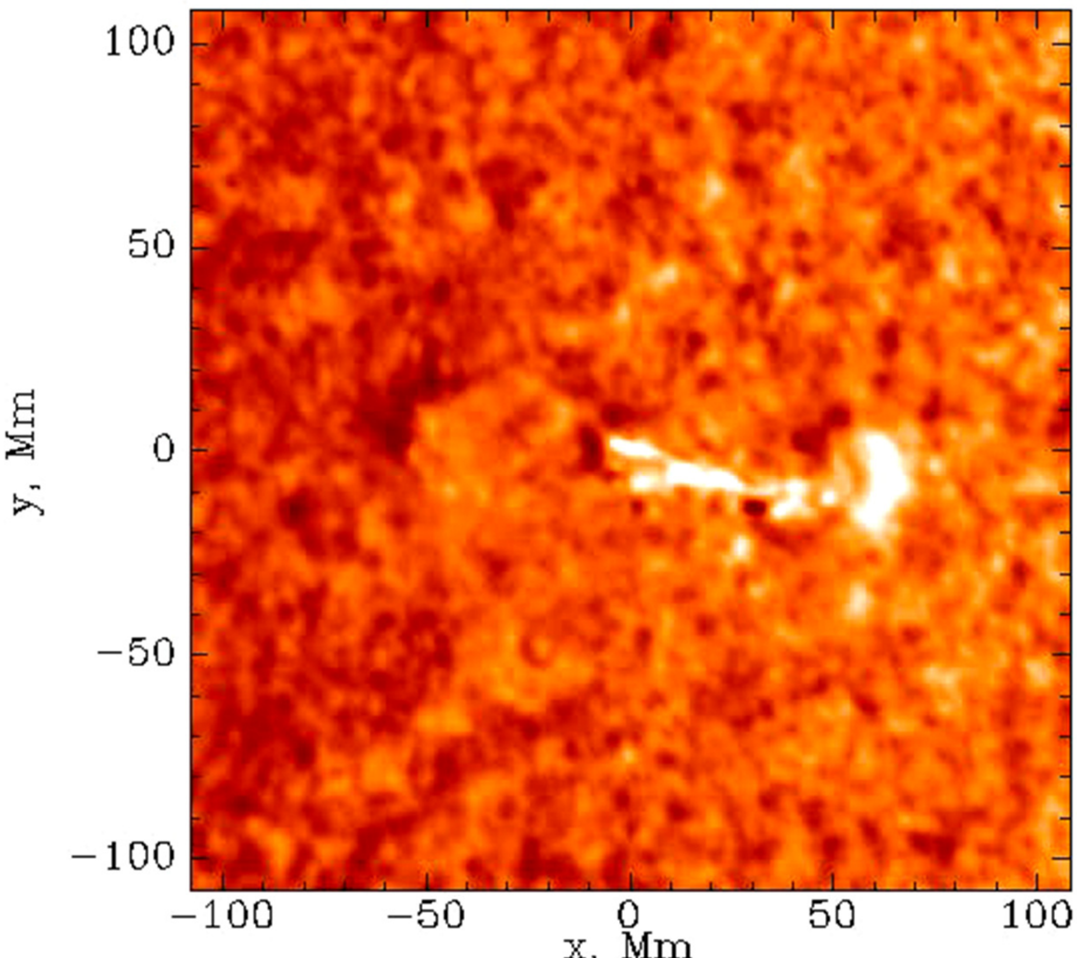
A sample of solar oscillations observed as a function of time and position on the disk

Velocity vs. time (horizontal axis) and position on the solar disk (vertical axis). The distance between the curves for adjacent points is equal to a velocity of 400 m/s.

Randomly excited solar oscillations

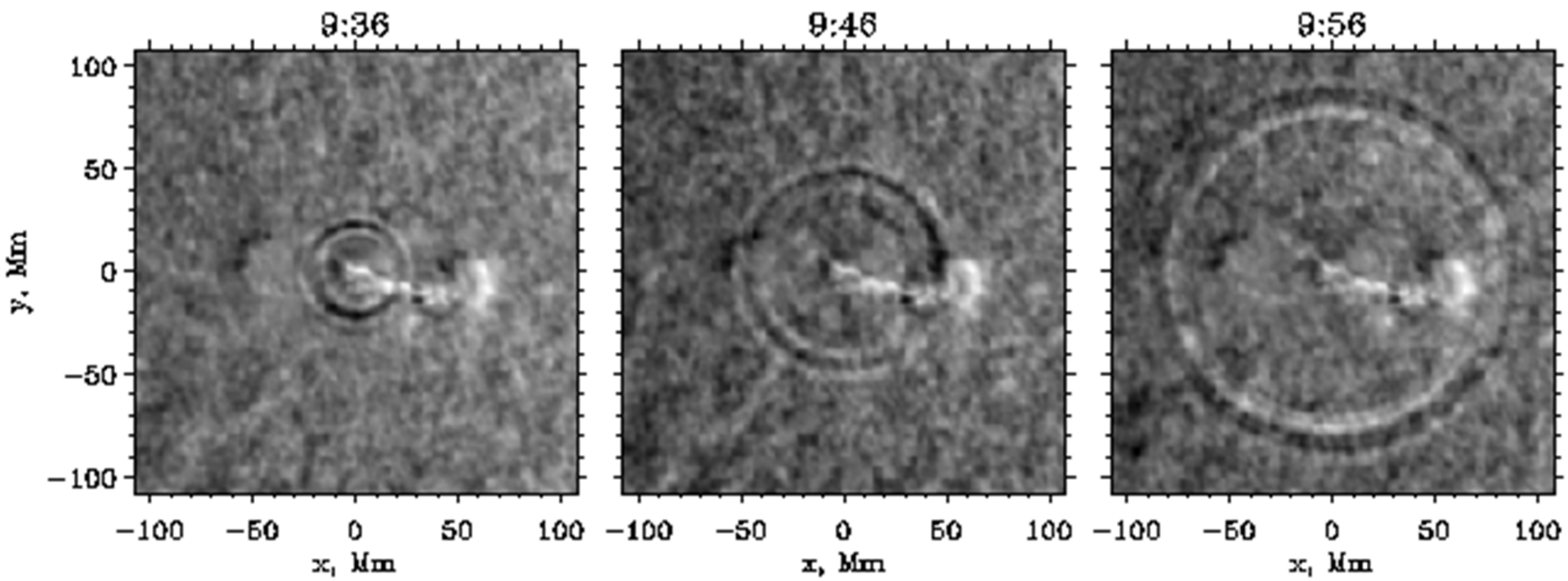


Seismic response to solar flares (sunquakes)



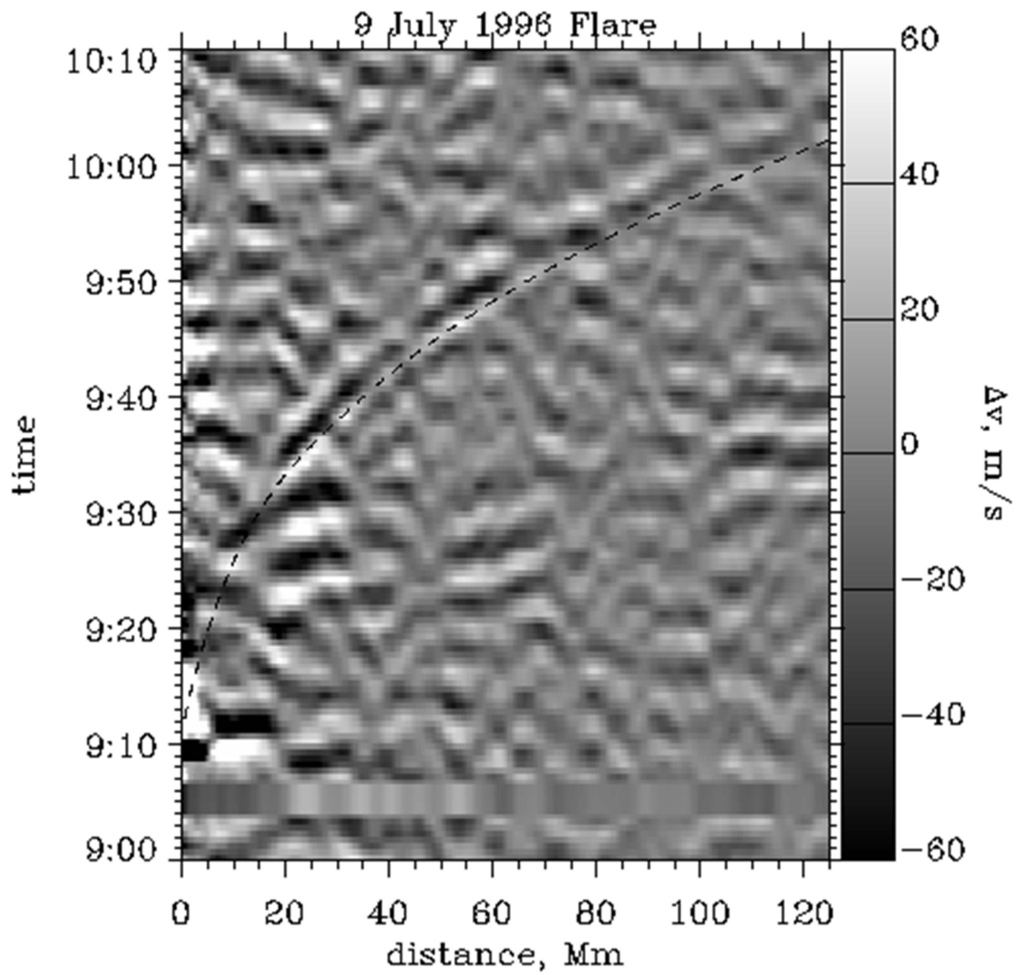
High-energy flare particles heat the solar chromosphere generating a shock propagating downward and hitting the surface.

Enhanced images of the flare ripples on the Sun's surface



Compare with water ripples

Time-distance diagram of the flare seismic response calculated by averaging the wave front over 360 degrees



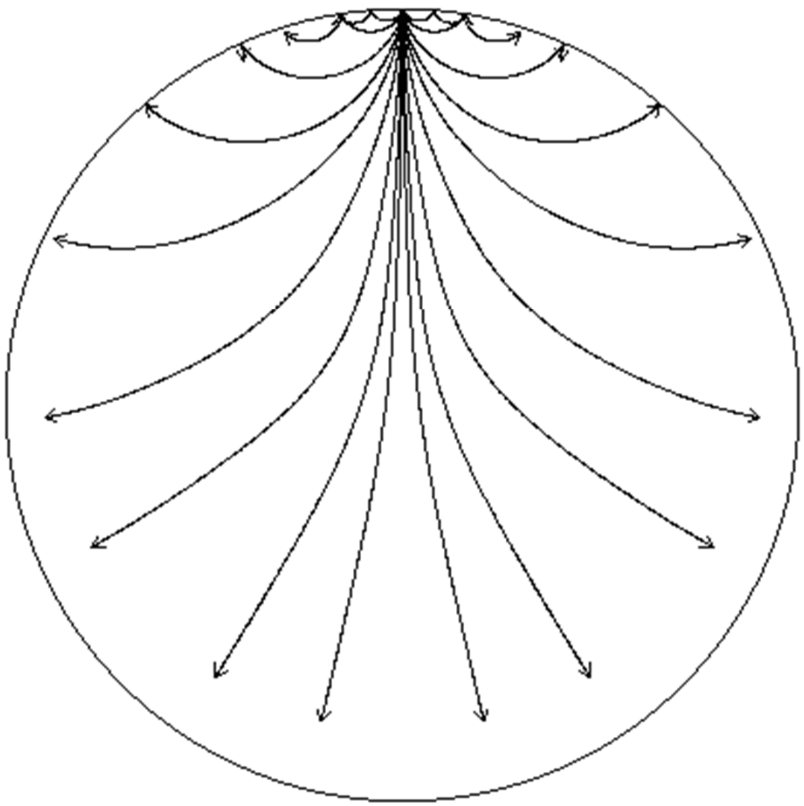
The propagation speed of the seismic wave:

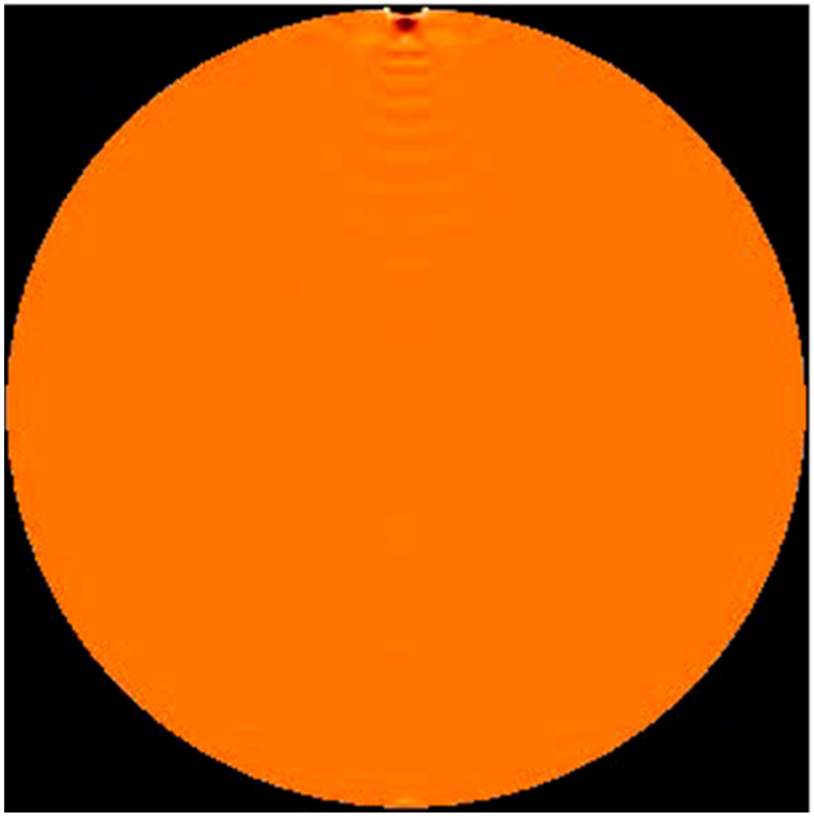
$$V=\delta(\text{distance})/\delta(\text{time})$$

increases with time from 10 km/s to 100 km/s because the sound speed increases with depth.

Propagation of acoustic waves on the Sun

The wave front on the surface accelerates because it is formed by acoustic waves propagating through the solar interior where the sound speed is higher.

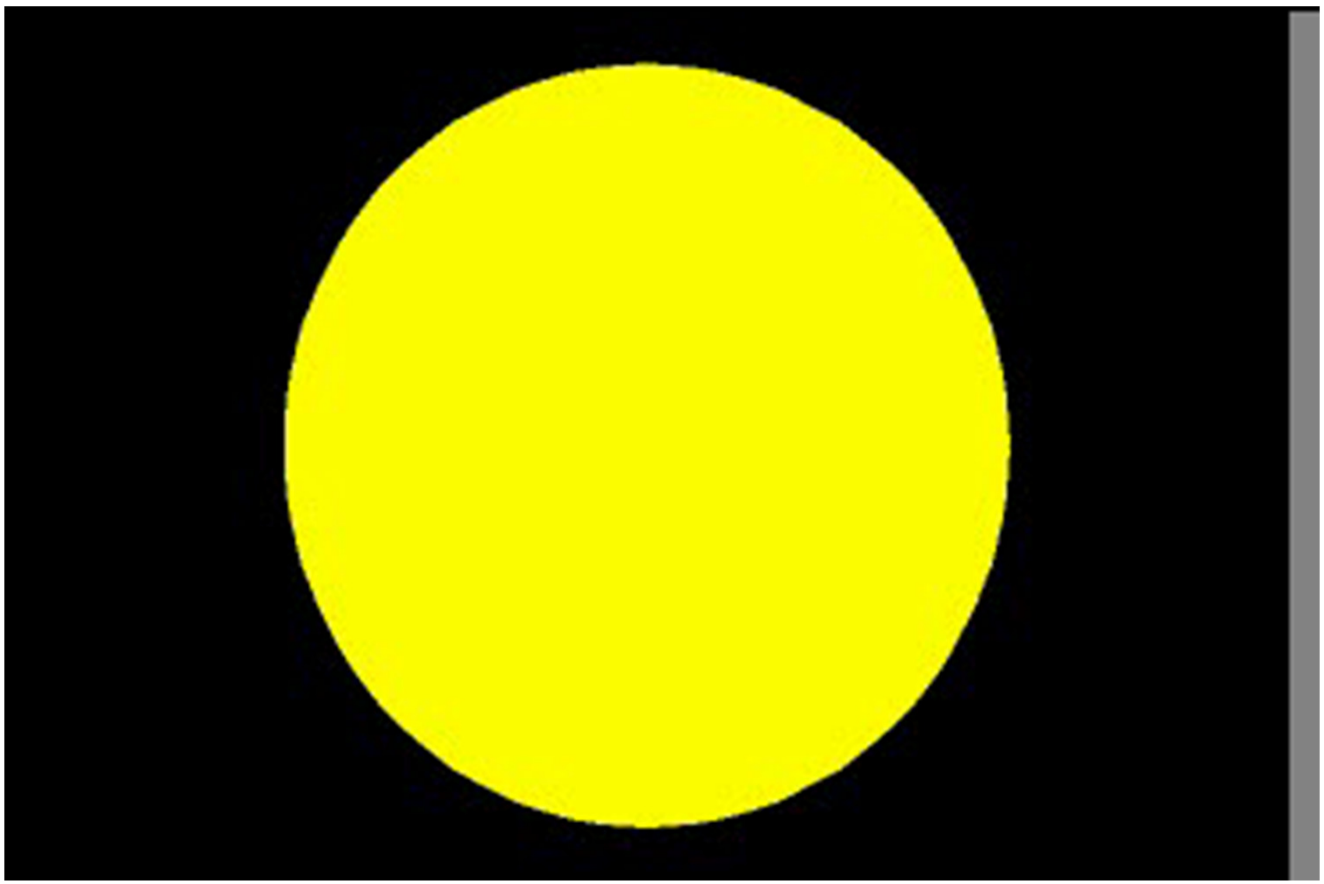




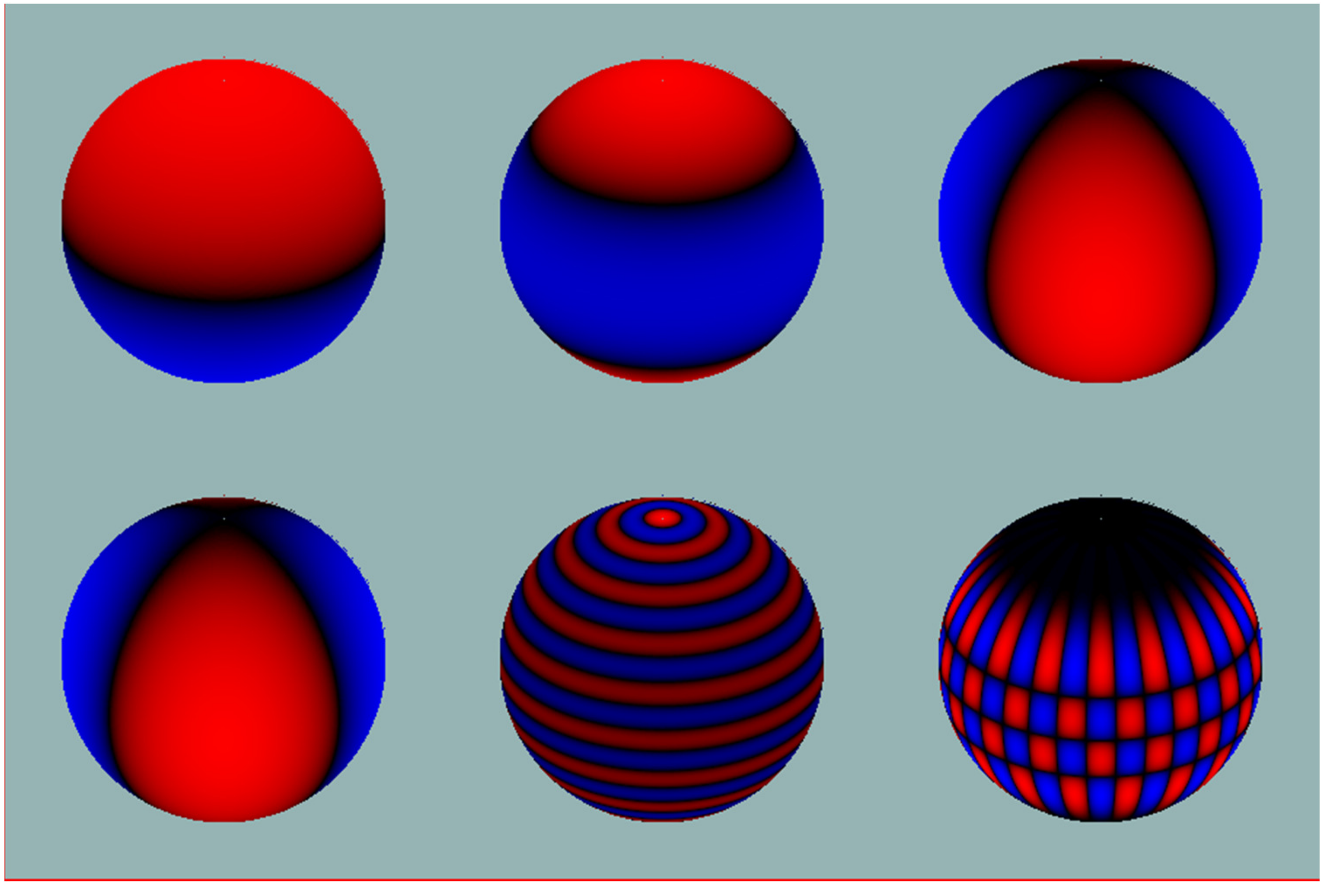
The ray paths are perpendicular to the wave fronts.

The idea of helioseismology

- Measure travel times τ or resonant frequencies ω of acoustic waves and infer the internal properties, e.g. $c_s(r)$ - sound speed

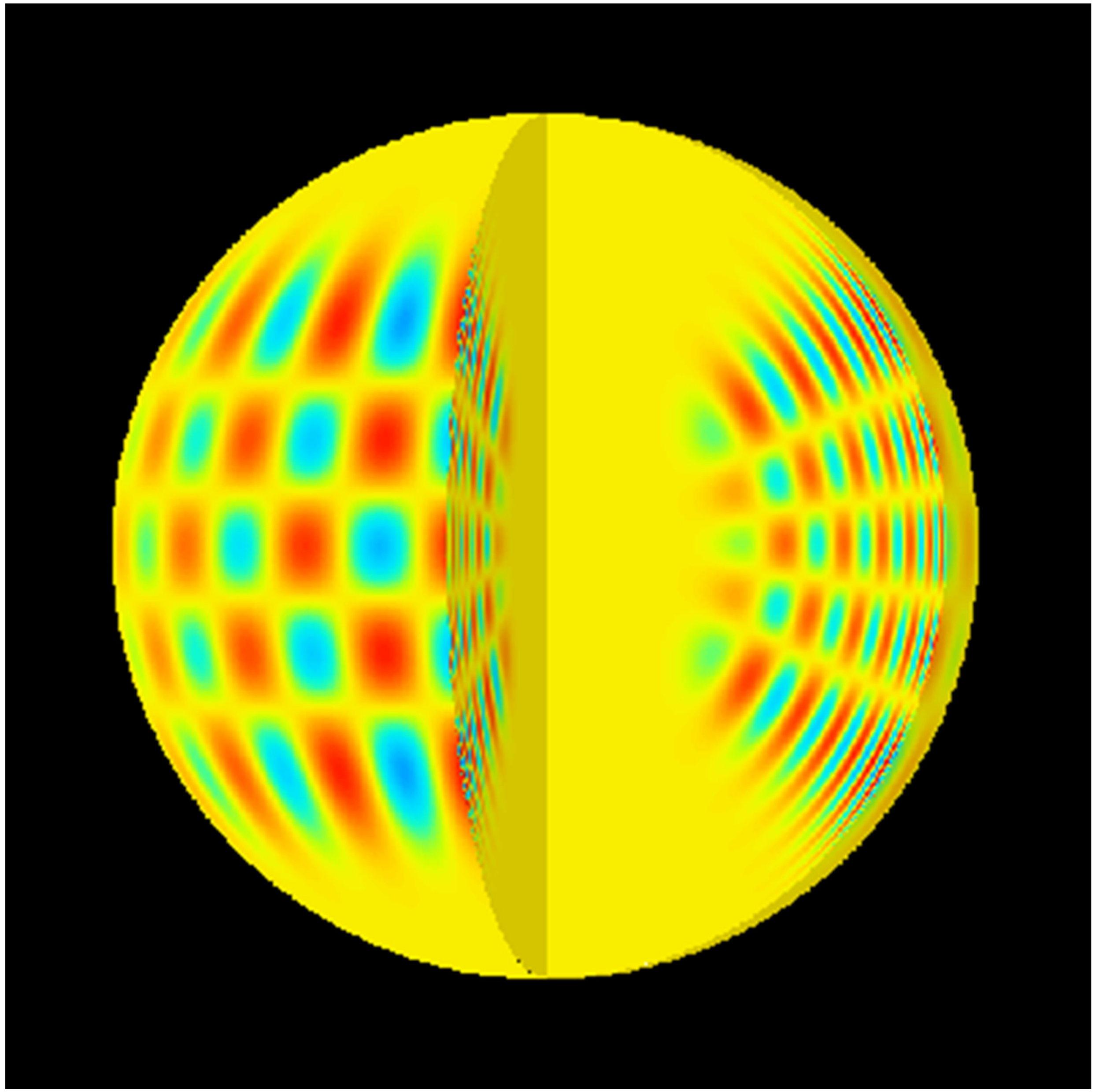


Basic properties of oscillations



- Behave like spherical harmonics: $P_l^m(\cos\theta)\cos(m\phi - \omega t)$
- $k_h = 2\pi / \lambda_h = [l(l+1)]^{1/2}/r$
- Form resonant wave patterns in the interior – normal modes

Normal Mode of Solar Oscillations



l=20, m=16

Power Spectrum

Velocity of oscillations $v(x,y,t)$ can be represented in terms of its Fourier components:

$$a(k_x,k_y,\omega)=\iiint v(x,y,t)e^{i(k_x x+k_y y+\omega t)}dxdydt,$$

where k_x and k_y are components of the wave vector, ω is the frequency.

The power spectrum is: $P(k_x,k_y,\omega)=a^*a,$ where a^* is complex conjugate.

If there is no preference in the direction of the wave propagation then P depends on two variables, the horizontal wavenumber $k_h=\sqrt{k_x^2+k_y^2}$, and frequency.

In the spherical coordinates, θ,ϕ:

$$a(l,m,\omega)=\iiint v(\theta,\phi,t)Y_l^m(\theta,\phi)e^{i\omega t}d\theta d\phi dt,$$

where $Y_l^m(\theta,\phi)$ is a spherical harmonic of **the angular degree** l **and angular order** m.

Degree l gives the total number of node circles on the sphere; order m is the number nodal circles through the poles.

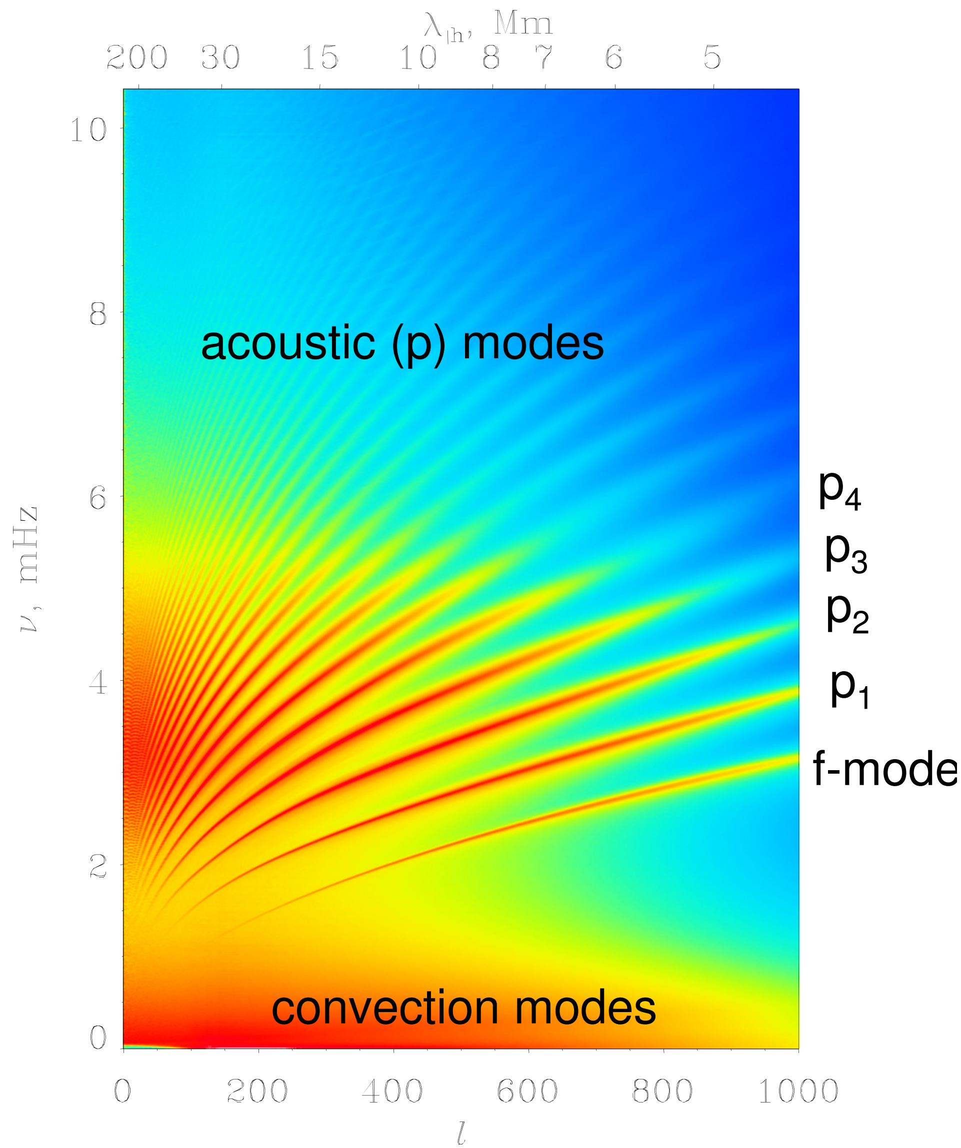
The power spectrum in this case is:

$$P(l,m,\omega)=a^*a.$$

For a spherically symmetrical star, P depends only on l and ω. The power spectrum is ‘degenerate’ with respect of angular order m. In this case the analog of the horizontal wavenumber is: $k_h=\dfrac{\sqrt{l(l+1)}}{R}$, where R is the solar radius.

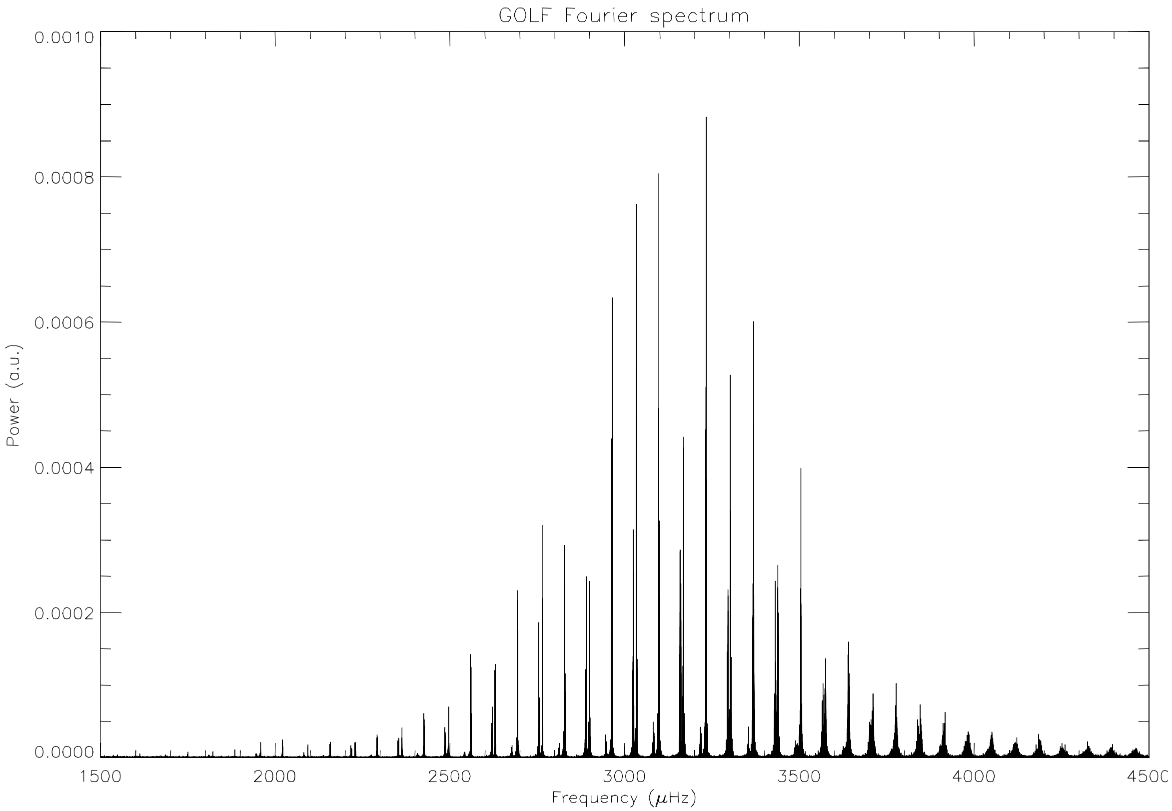
Oscillation power spectrum

- Spherical harmonic transform – oscillation signal is represented in terms of spherical harmonics of angular degree l.



Low-Degree (Global) Modes

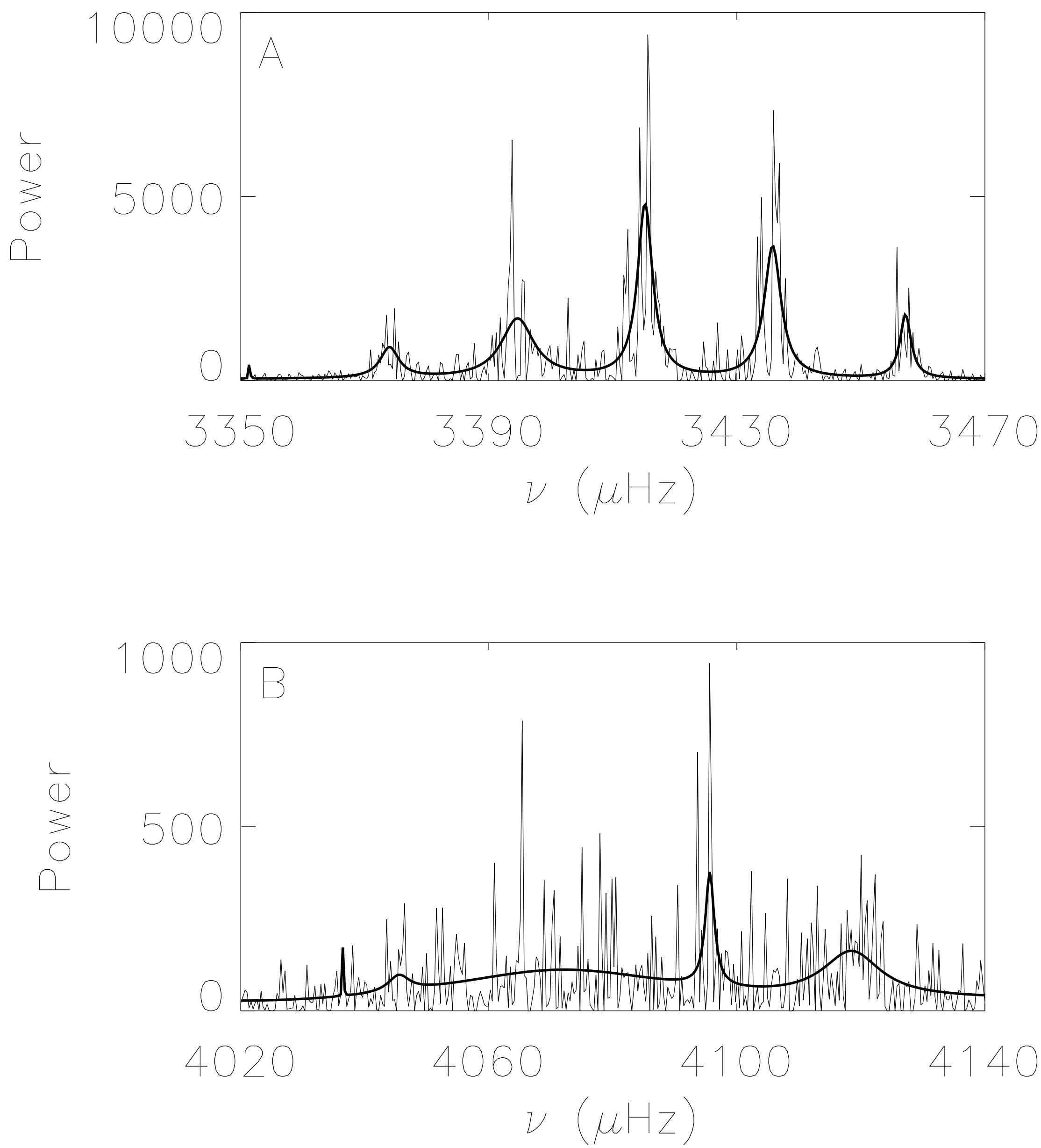
When the Sun is observed as a star (integrated whole-disk Doppler-shift measurements) the power spectrum consists only of low-degree p-modes of $l = 0, 1, 2$ and 3.



The distance between main peaks in the power spectrum is about 68 μHz. The corresponding time: $1/(68 \cdot 10^{-6}) = 245$ min is the travel time for acoustic waves propagate through the center of the Sun to the far side and come back. The low-degree mode provide information about physical conditions of the solar core.

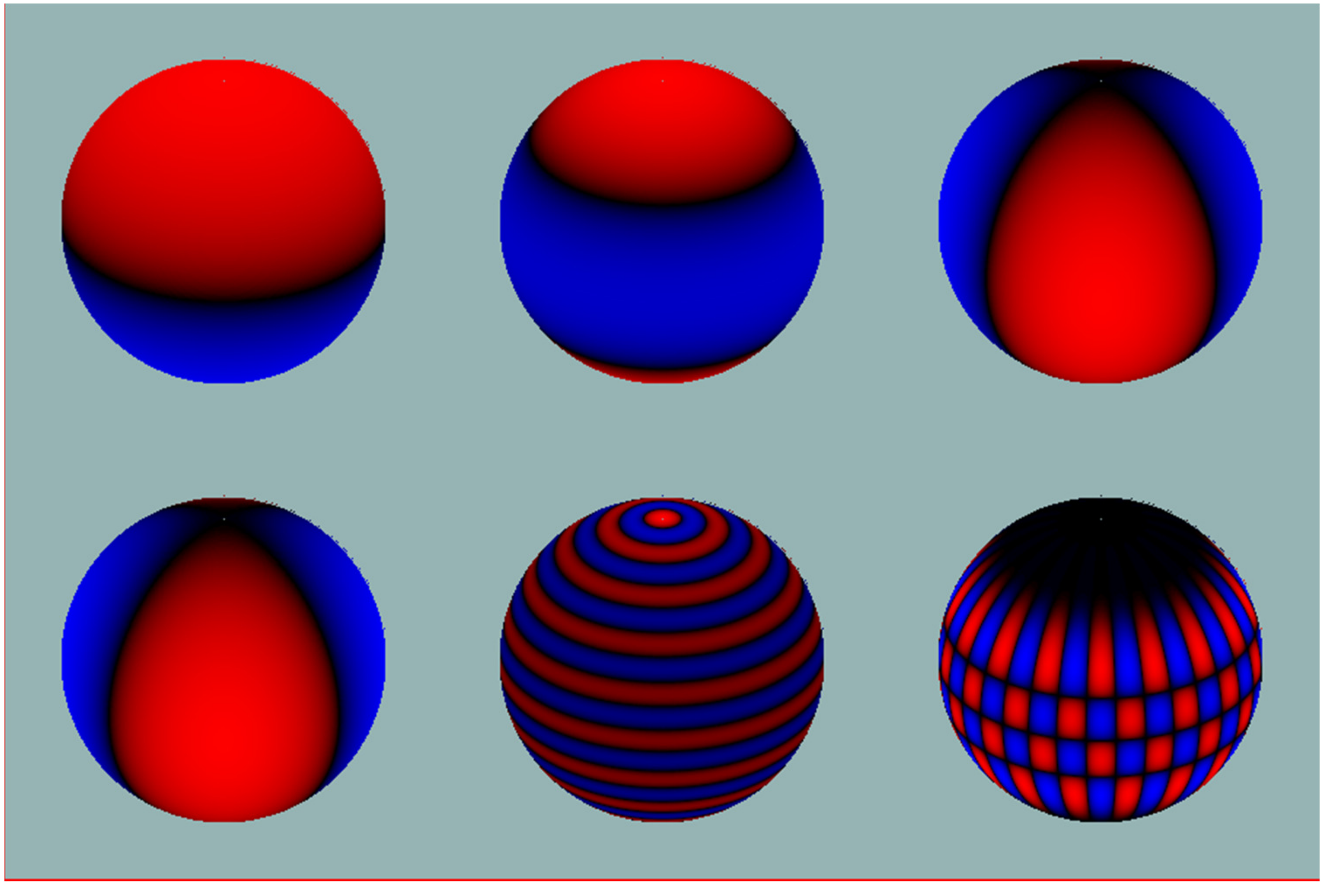
This figure is a Fourier spectrum of the longest continuous GOLF time series (805 days). GOLF is an instrument on SOHO that measures the oscillations in the line-of-sight velocity of the solar photosphere from the whole Sun. These oscillations appear at precise frequencies, visible as sharp peaks in this spectrum, mainly around 3mHz, corresponding to periods about 5min.

Excitation of Solar Oscillations



Power spectra of A) $l = 50, m = -32, n = 12$ and B) $l = 50, m = 0, n = 16$.

Solar oscillations are randomly excited by turbulent convection. The random excitation function appears as multiplicative noise in the power spectra. This represents the most serious problem for measuring mode frequencies. This figure shows examples of good and poor fits of an oscillation model to the power spectra.



Cyclic frequency $\nu = \frac{\omega}{2\pi}$ is often used as frequency variable.

Because only a hemisphere of the Sun is observed in the power spectrum for a given mode of target l, m beside peaks corresponding to this mode peaks of other modes appear (so-called 'mode leaks'). The spherical harmonics are not orthogonal on a hemisphere.

Rotational frequency splitting

The modes with $m \neq 0$ represent azimuthally propagating waves. The modes with $m > 0$ propagate in the direction of solar rotation and, thus, have higher frequencies in the inertial frame than the modes $m < 0$ which propagate in opposite direction. As a result the modes with fixed n and l are split in frequency: $\Delta\nu_{nlm} = \nu_{nlm} - \nu_{nl0}$. Thus, the internal rotation is inferred from splitting of normal mode frequencies with respect to the azimuthal order, m .

$$\vec{\xi} \propto e^{i\omega t} Y_l^m(\theta,\phi) = C P_l^m(\theta) e^{im\phi + i\omega t}$$

- displacement of the solar surface in solar modes

$$\nu = \omega / 2\pi$$

– ν is cyclic frequency, measured in Hz
- The oscillation period is 1/ν (in sec, min, etc).

ω is the angular frequency, measured in rad/s

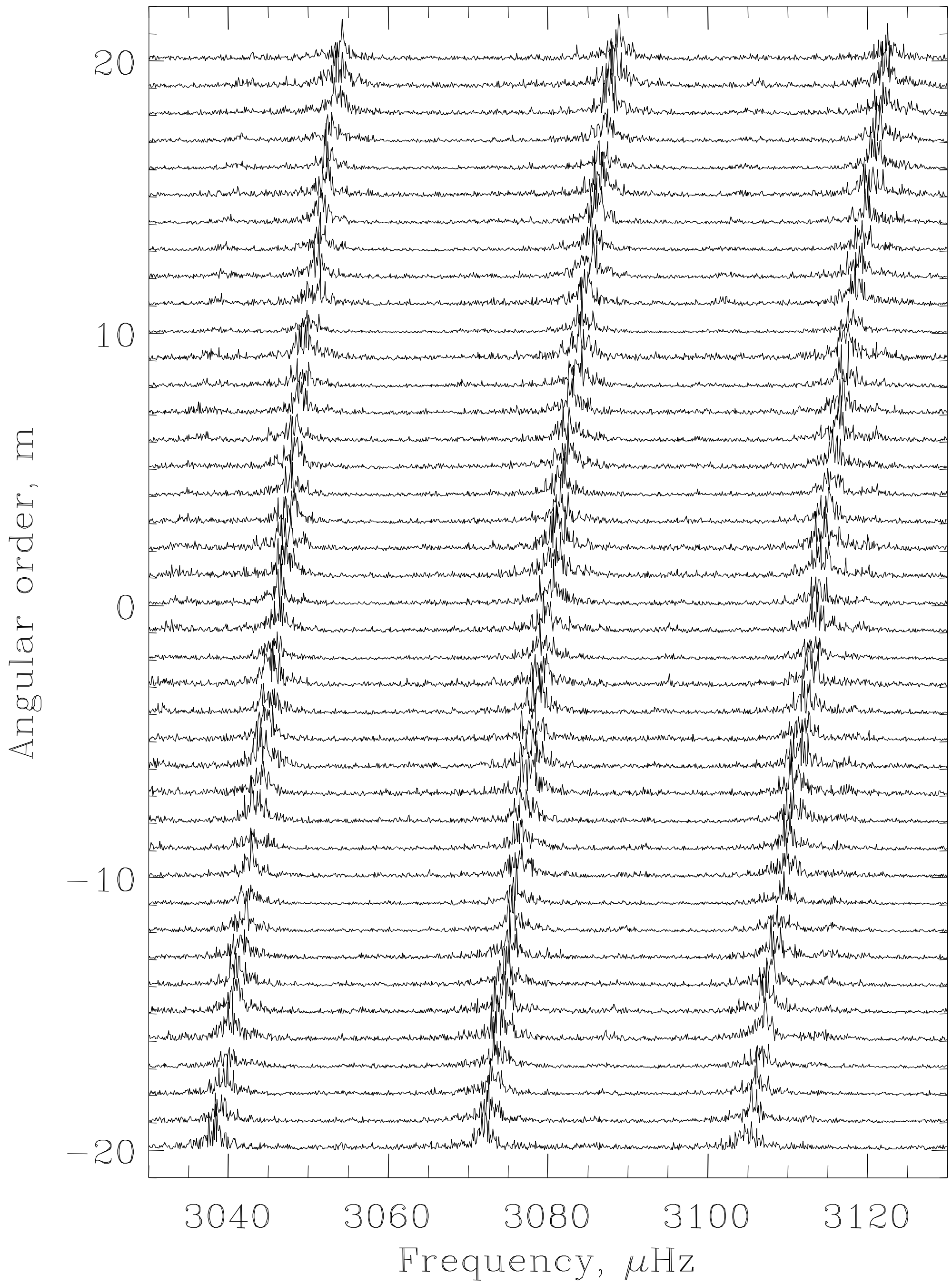


Illustration of the frequency shift due to the solar rotation

Typical power spectra of solar oscillation data from the MDI instrument on SOHO. Each horizontal curve shows three lines of the power spectrum for different azimuthal order m with radial order $n = 15$ and angular degree $l = 19, 20$, and 21 (from left to right). The slope of the modal lines is due to the rotational frequency shift: prograde modes with positive m have higher frequencies than retrograde modes with negative m.