14. Solar MHD

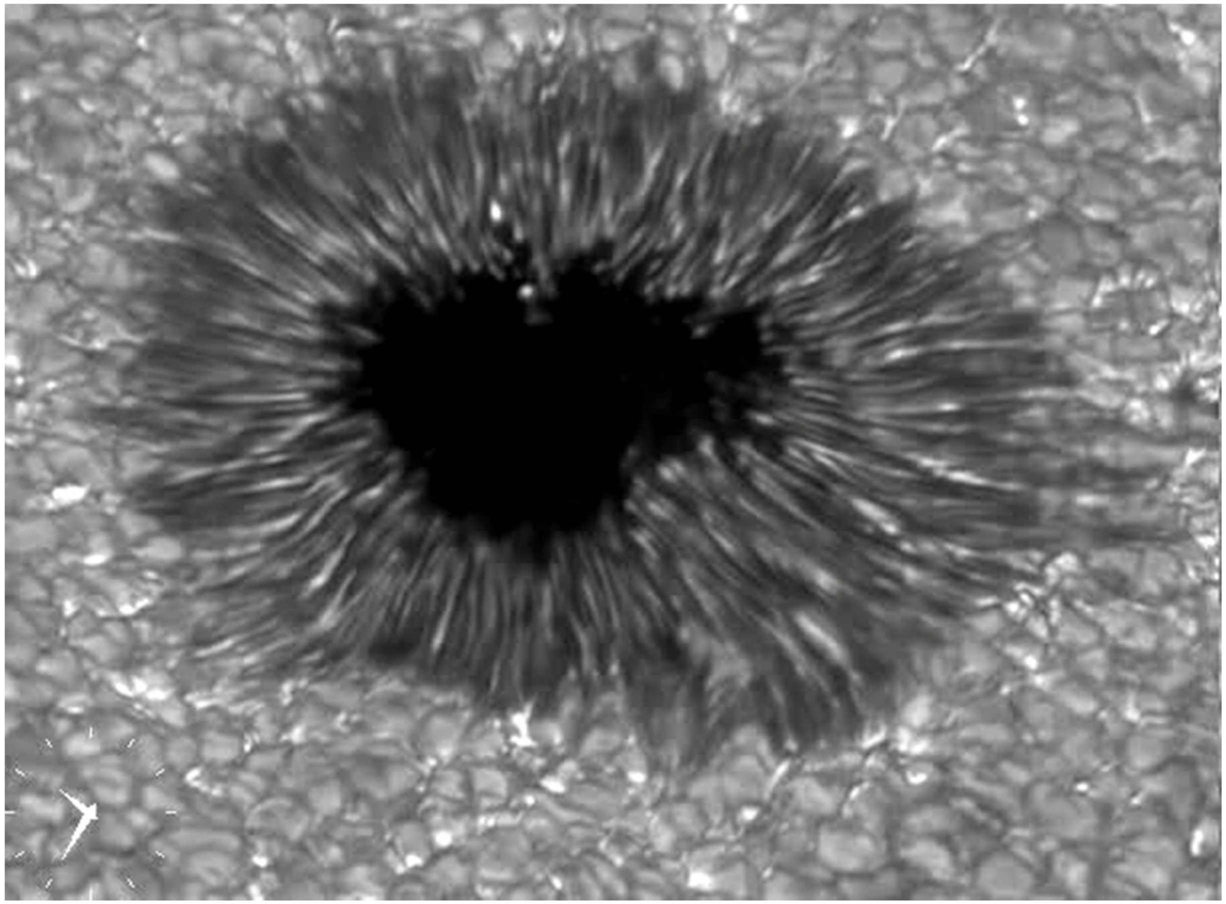
Solar MHD

- Particle Motion in Electric Field
- Magnetic Effects
- Ohm’s Law
- MHD Equations
- Magnetic Field Diffusion
- Frozen Magnetic Flux Approximation
- Magnetic Forces
- MHD Waves
- *Magnetohydrodynamics movie:* https://www.youtube.com/watch?v=QArcTylNooQ

Examples of MHD dynamics

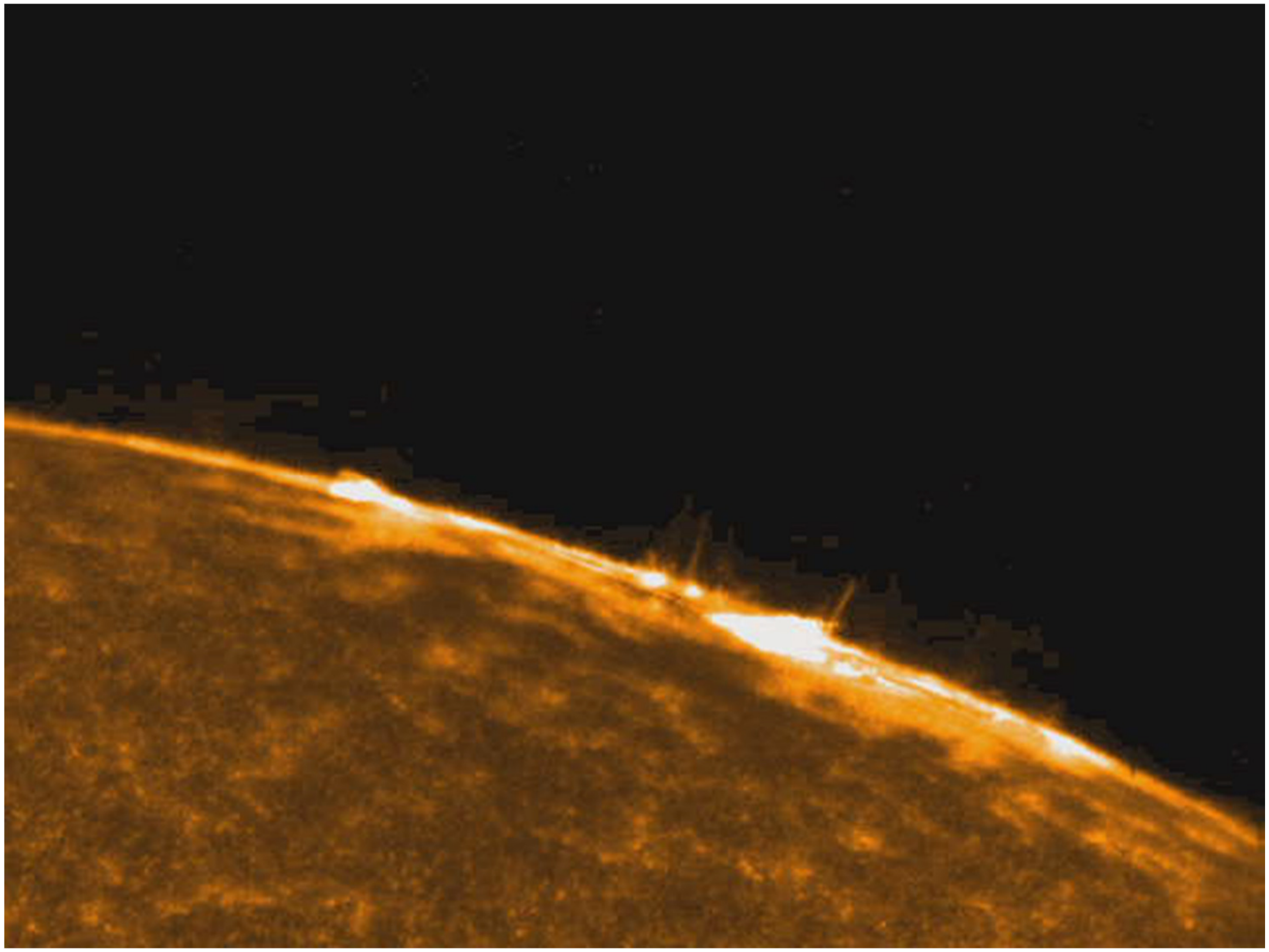
- Sunspots
- Flares
- Coronal mass ejections
- Oscillations and waves

Sunspots



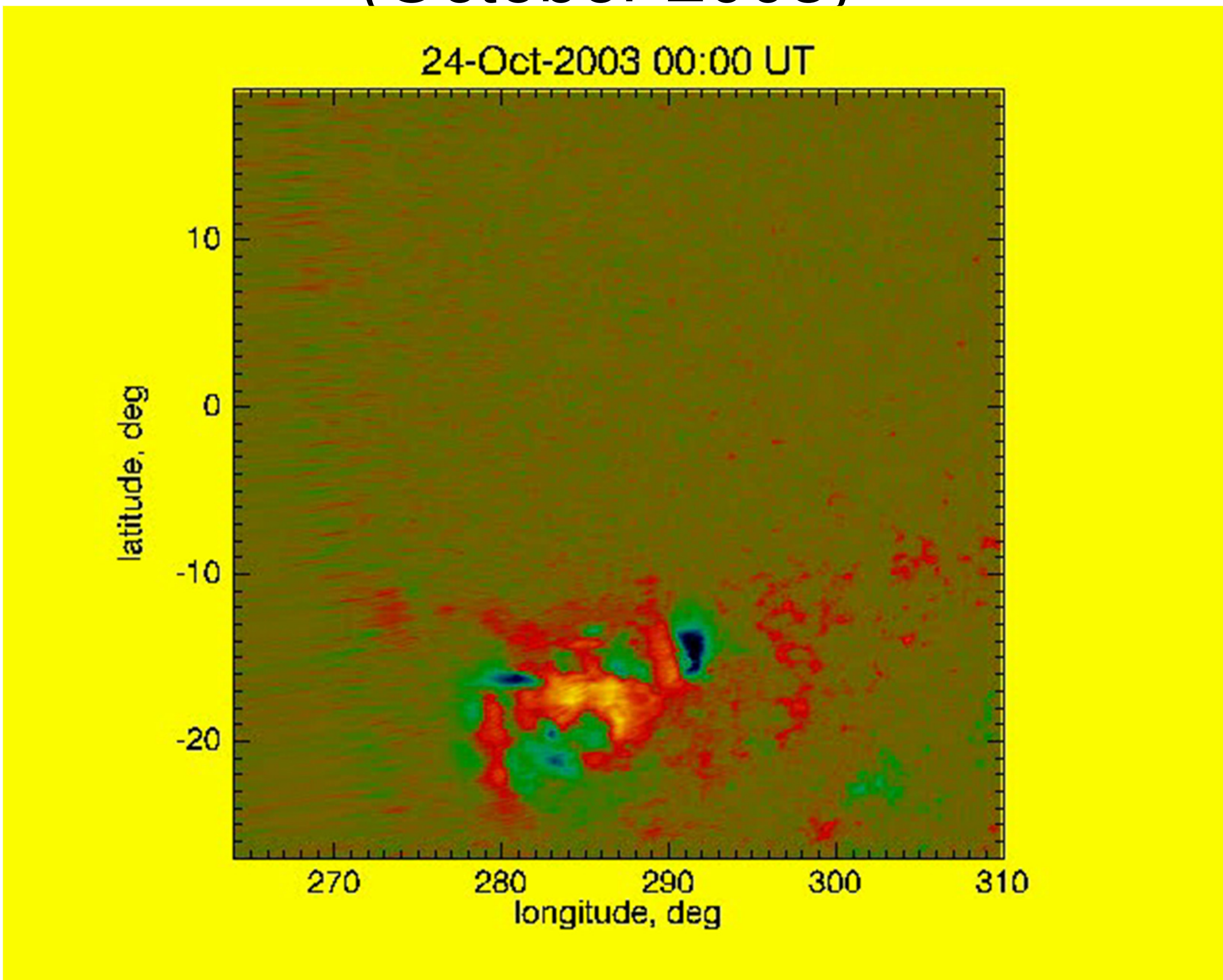
Size: 10-20 Mm, Lifetime: 1 Month

Flares

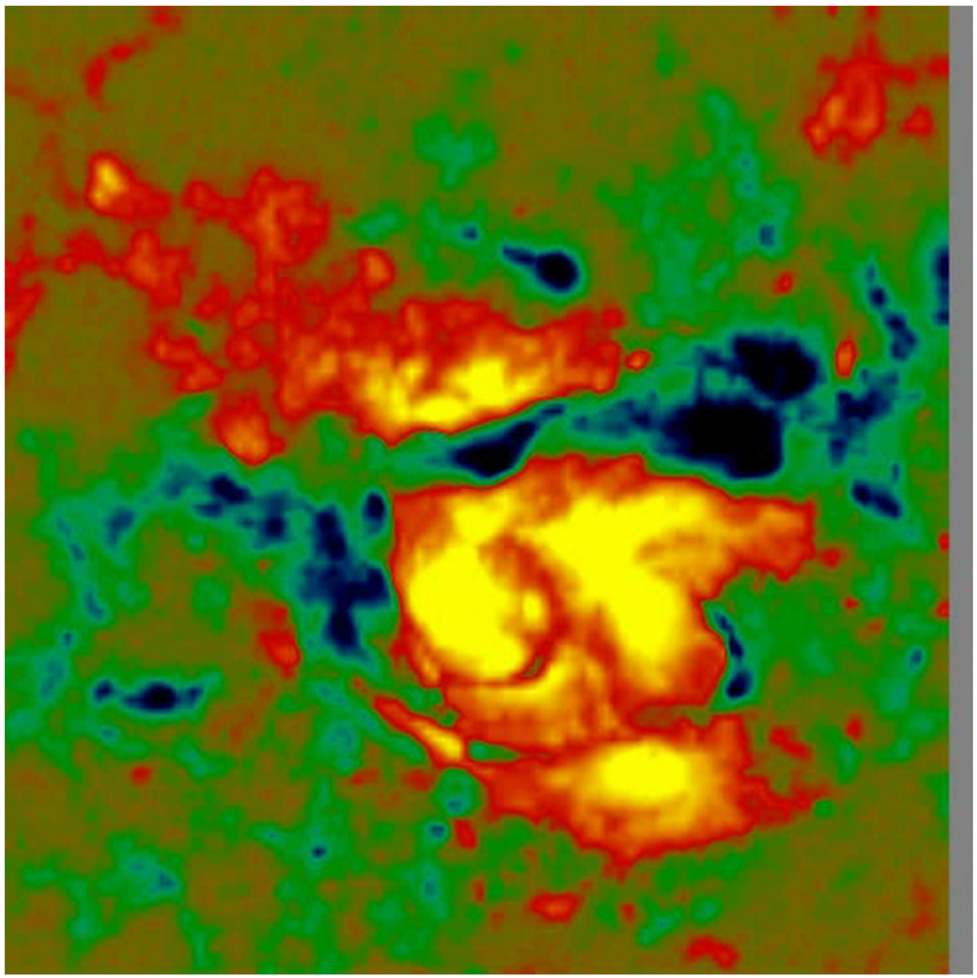


Size: 10-100 Mm, lifetime: 100-1000 sec

Magnetic field of Active Region 10486 (October 2003)



Magnetic field changes in X10 flare, Oct. 29, 2003, 20:37 UT



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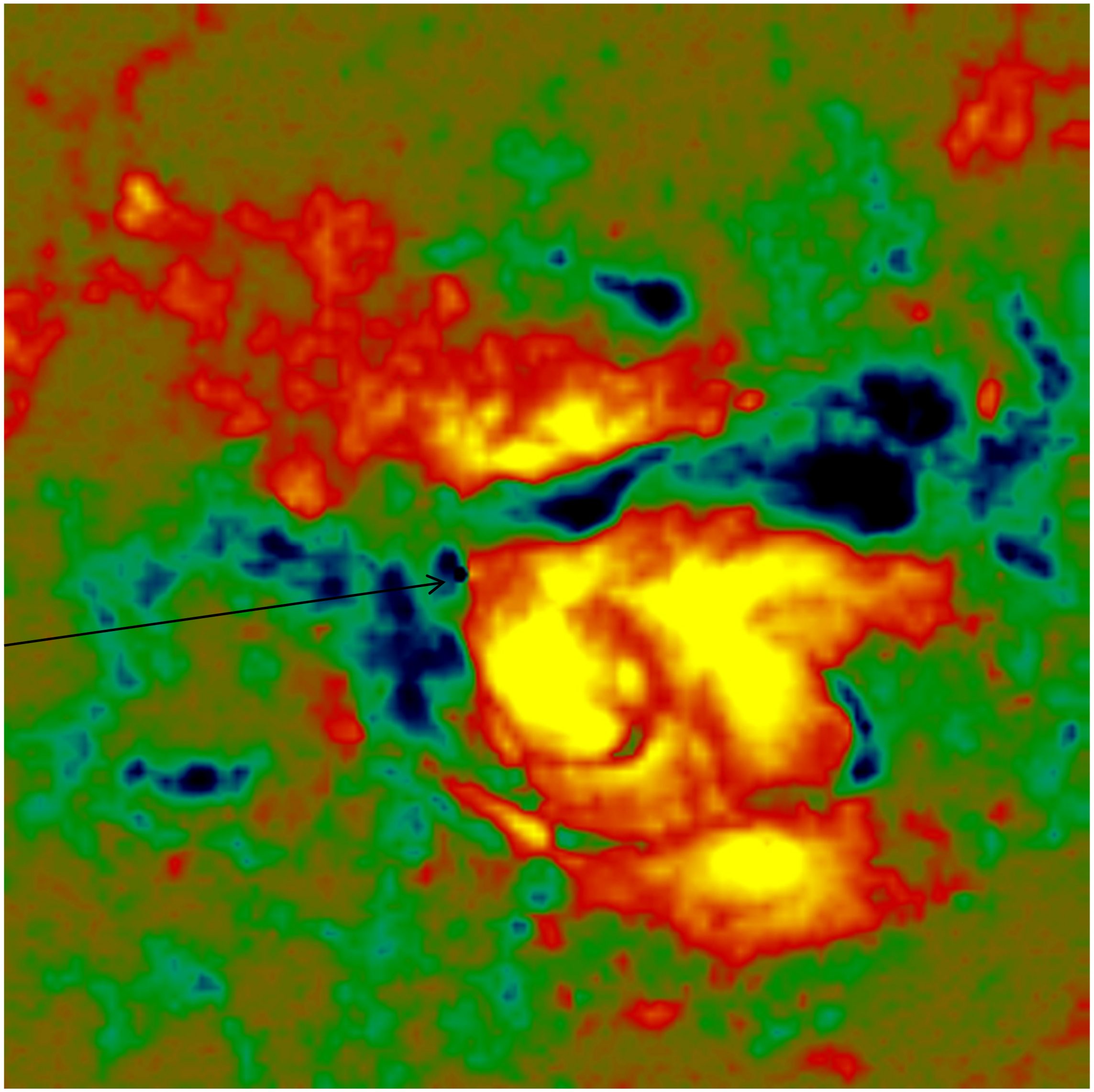
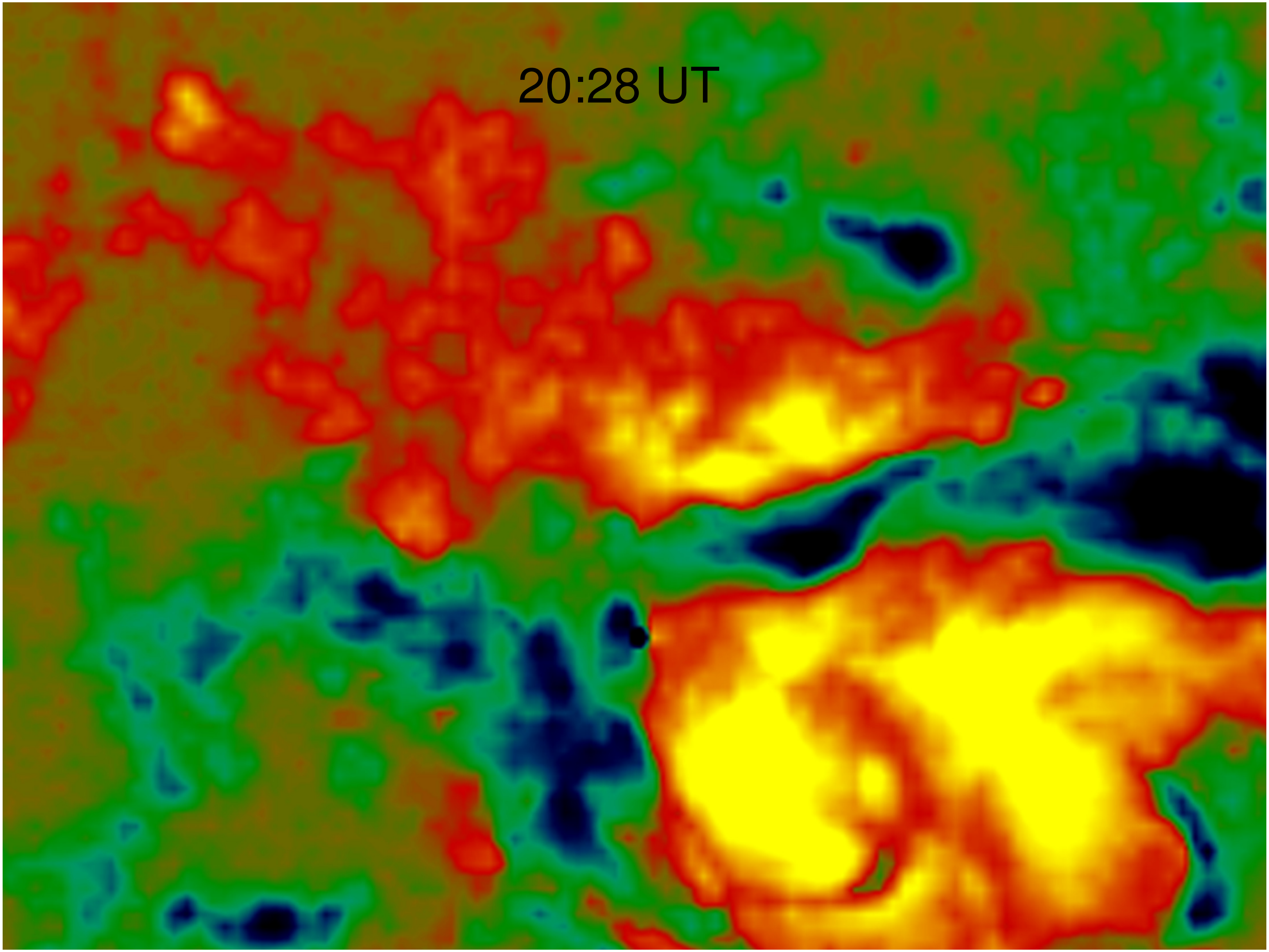


Flare energy release is associated with magnetic neutral lines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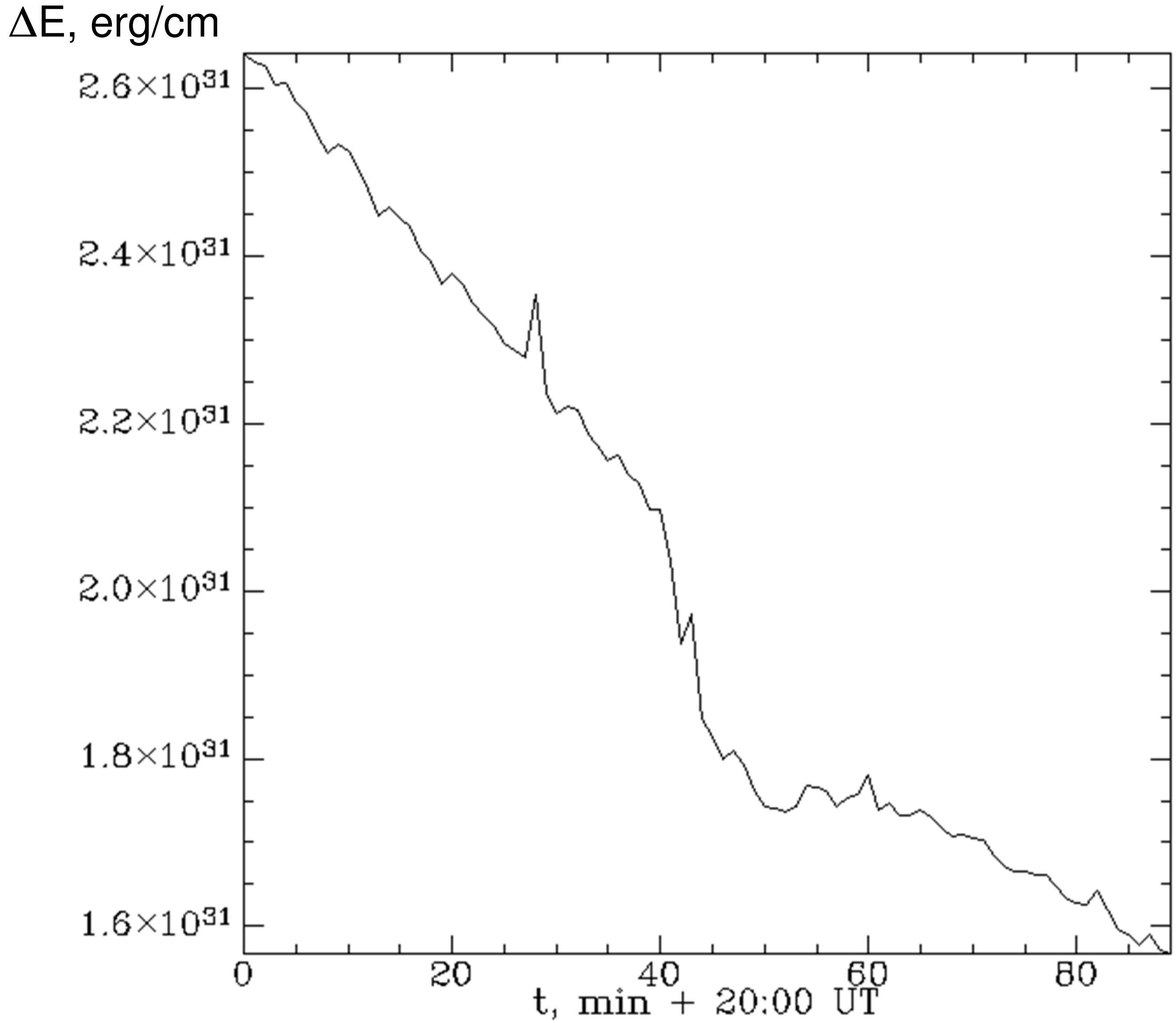
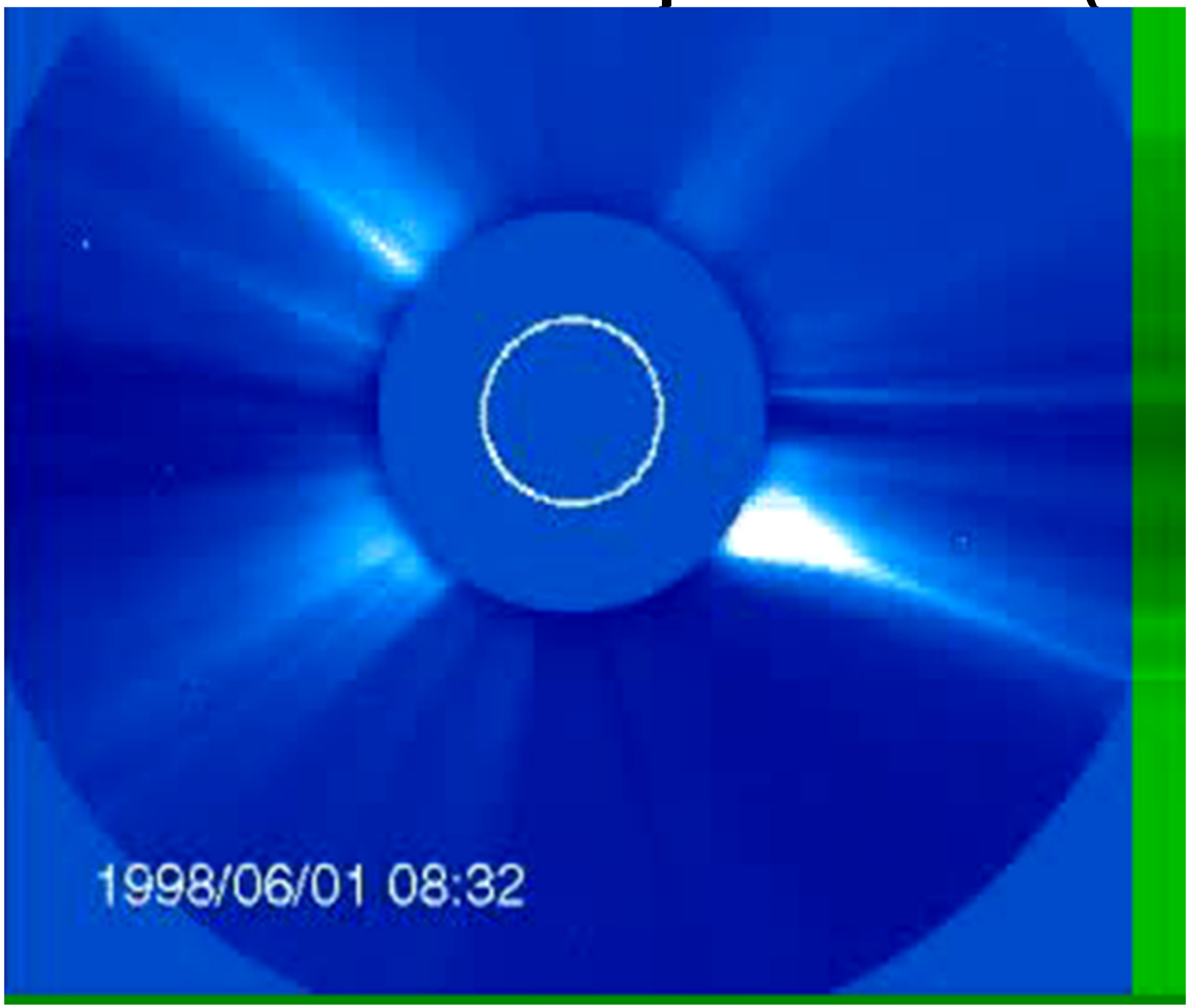
Change of the magnetic energy during the flare

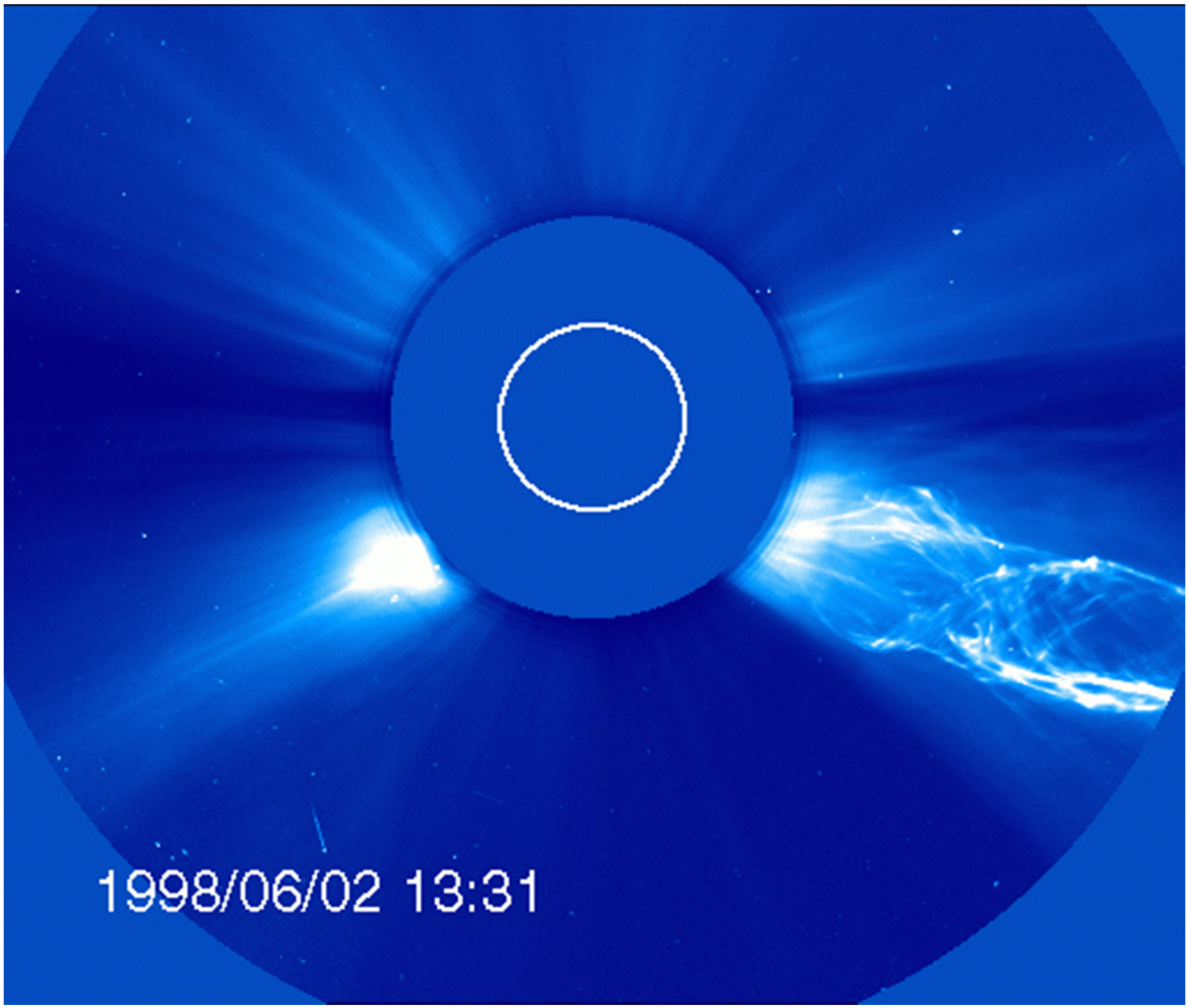


Coronal mass ejections (CME)

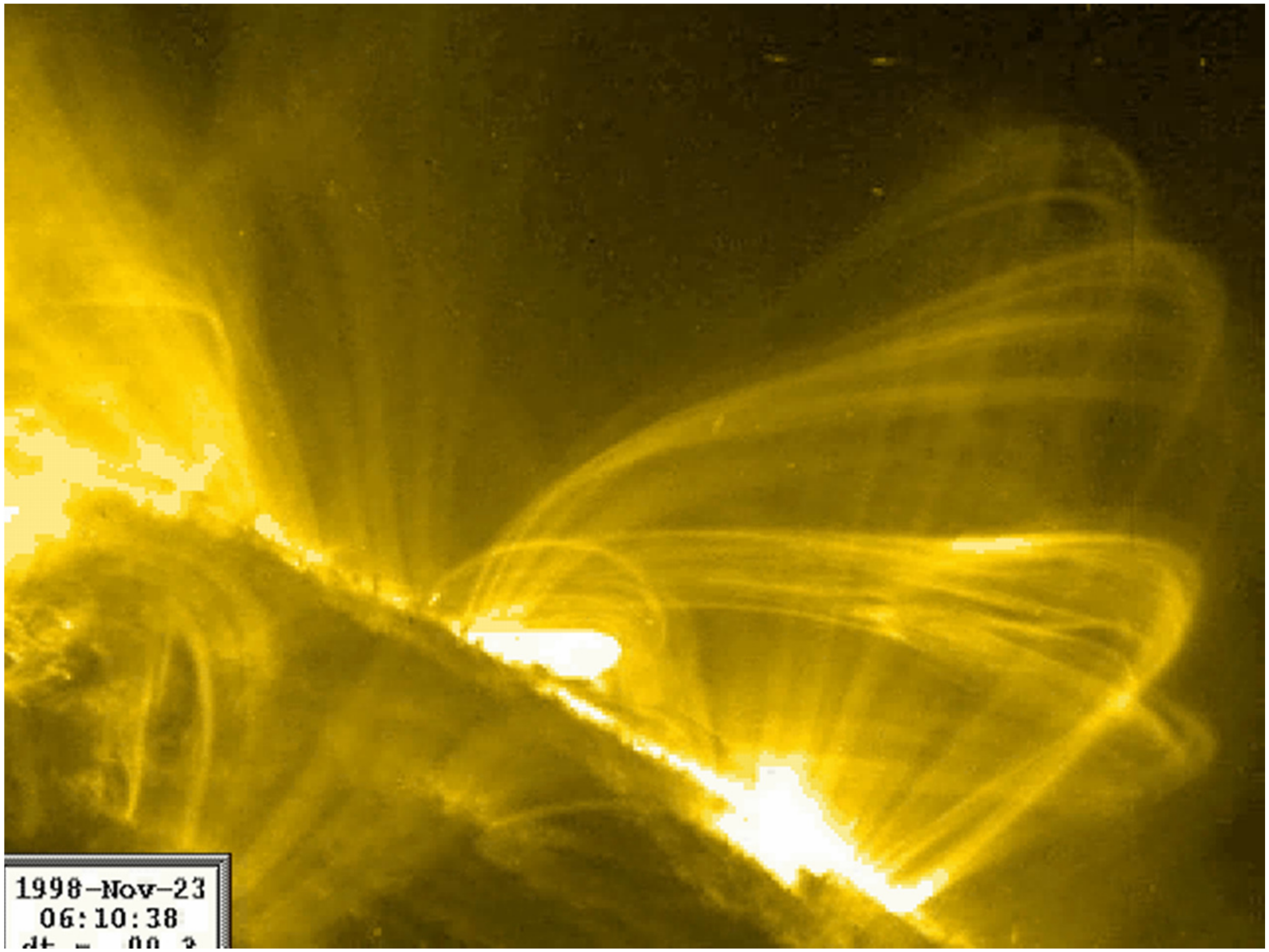


Size: 100 Mm, eruption time: 1000 sec

Coronal mass ejections show twisted magnetic structures



Coronal loop oscillations represent various types of MHD waves



Size: 100 Mm, period: 300 sec

Particle Motion in Electric Field

Consider particle motions in a fully-ionized plasma in an electric field, E:

$$m\frac{d\mathbf{v}}{dt} = -\frac{m\mathbf{v}}{\tau} + e\mathbf{E},$$

where τ is a characteristic time of the loss of momentum due to collisions.

Then, a mean stationary velocity is:

$$\left\langle m\frac{d\mathbf{v}}{dt} \right\rangle = -\left\langle \frac{m\mathbf{v}}{\tau} \right\rangle + e\mathbf{E} = 0,$$

$$< \mathbf{v} >= \frac{e\tau\mathbf{E}}{m}.$$

Particle Motion in Electric Field

$$<\mathbf{v}> = \frac{e\tau\mathbf{E}}{m}.$$

We assume that plasma is fully ionized $n_e = Zn_i, \quad n_i = n$, Z is the charge number of the ions. Then, the corresponding electric current density is:

$$\mathbf{j} = Zen <\mathbf{v}> = \frac{Ze^2\tau n}{m}\mathbf{E},$$

or

$$\mathbf{j} = \sigma\mathbf{E},$$

where $\sigma = \dfrac{Ze^2\tau n}{m}$ is electrical conductivity.

Coulomb collisions.
Collision cross-section.

The characteristic change of momentum is equal to the characteristic Coulomb force at the characteristic collision distance, d :

$$\frac{mv}{\tau} \simeq \frac{Ze^2}{d^2}.$$

Since $\tau \simeq d/v$ then the characteristic collision distance (the distance at which the particle looses its momentum) is:

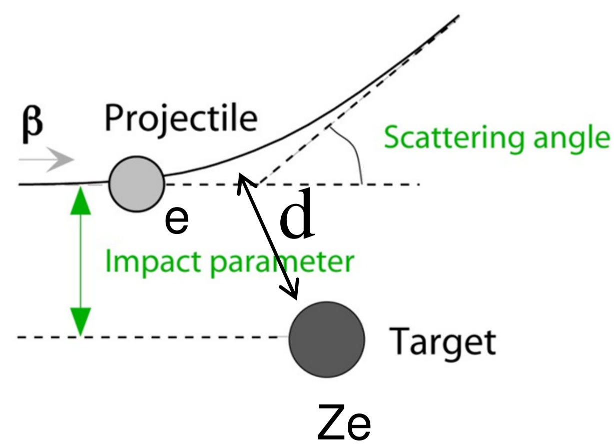
$$d \simeq \frac{Ze^2}{mv^2}.$$

The corresponding collision cross-section is:

$$\Sigma \sim d^2 \sim \frac{Z^2e^4}{m^2v^4},$$

or more precisely $\Sigma = \frac{Z^2e^4}{m^2v^4}\Lambda,$

where $\Lambda \sim 10$ is "the Coulomb logarithm".



Coulomb collisions.

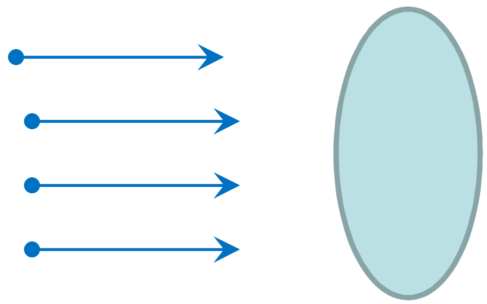
Collision frequency.

The collision cross-section is:

$$\Sigma = \frac{Z^2 e^4}{m^2 v^4}\Lambda,$$

particle density: n

particle flux: nv



The collision frequency is: $\nu_{collision} = nv\Sigma$.

The corresponding characteristic collision time: $\tau = \frac{1}{nv\Sigma} = \frac{m^2 v^3}{nZ^2 e^4 \Lambda}$.

Electrical conductivity of plasma

The characteristic collision time:

$$\tau = \frac{1}{nv\Sigma} = \frac{m^2 v^3}{nZ^2 e^4 \Lambda}.$$

Then, electrical conductivity σ: $\sigma = \frac{Ze^2 \tau n}{m}$

$$\sigma = \frac{Ze^2 n m^2 v^3}{mnZ^2 e^4 \Lambda} \simeq \frac{(kT)^{3/2}}{m^{1/2} Ze^2 \Lambda},$$

where the particle speed, v, is estimated using the definition of the plasma temperature: $kT \simeq mv^2$.

A practical formula for the collision cross-section in fully ionized hydrogen plasma:

$$\Sigma = \frac{3\times 10^{-13}}{T_{\mathrm{eV}}^2},\ \mathrm{cm}^2,$$

where T_{eV} is the plasma temperature in eV. 1 eV=11,600 K.

Run-away electrons. Dreicer field.

Now, return to the momentum equations:

$$\frac{dv}{dt} = -\frac{v}{\tau} + \frac{eE}{m} = -\frac{nZ^2e^4\Lambda}{m^2v^2} + \frac{eE}{m}.$$

When

$$E > \frac{nZ^2e^3\Lambda}{mv^2} \equiv E_D$$

then there is no stationary mean velocity; the particles continuously accelerates (**"run-away electrons"**). The critical electric field E_D is called **Dreicer field.**

Comment: The Dreicer field can be expressed in terms of Debye radius:

$$E_D \simeq \frac{nZ^2e^3\Lambda}{kT} \approx \frac{e}{\lambda_D}, \text{ where } \lambda_D^2 = \frac{kT}{4\pi Z^2ne^2} \text{ is the}$$

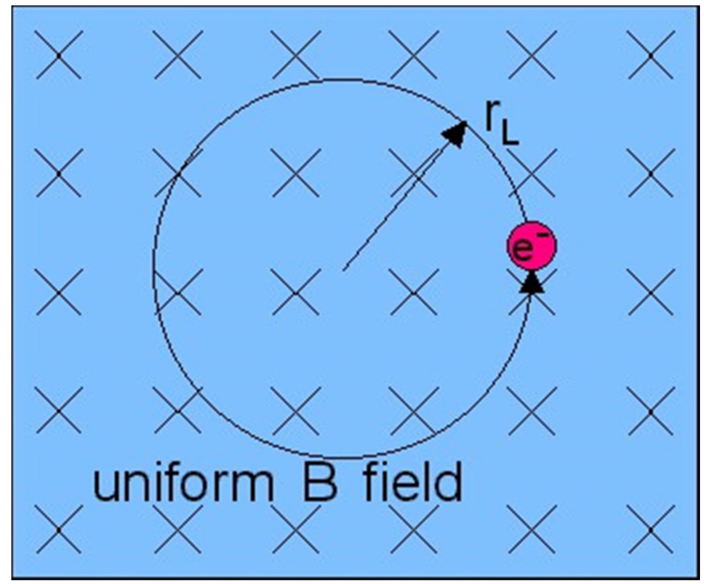
Debye (‘shielding’) length for electrons. That means that the Dreicer field is approximately the electric field of electron at the Debye-length distance from the charge.

Run-away electrons may contribute to coronal heating

Magnetic Effects

Consider motion of a charged particle perpendicular to the magnetic field lines. In this case the particle has a circular motion, so that the Lorentz force is balance by centrifugal force:

$$\frac{ev_\perp B}{c} = \frac{mv_\perp^{\ 2}}{r}.$$



The radius of this motion:

$$r = \frac{mv_\perp c}{eB}$$

is called the **Larmor radius.**

The corresponding frequency of the circular motion

$$\omega_L = \frac{v_\perp}{r} = \frac{eB}{mc}$$

is the **Larmor frequency.**

Ohm's Law

Consider a charged particle in both electric and magnetic fields. The equation of motion is:

$$\frac{d\mathbf{v}}{dt} = -\frac{\mathbf{v}}{\tau} + \frac{e}{m}(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}),$$

then the mean stationary velocity is:

$$<\mathbf{v}> = \frac{\tau e}{m}(\mathbf{E} + \frac{1}{c} <\mathbf{v}> \times\mathbf{B}).$$

Using the definition for the electric current density we get:

$$\mathbf{j} = \sigma\mathbf{E} + \frac{e\tau}{mc}(\mathbf{j}\times\mathbf{B}) = \sigma\mathbf{E} + \frac{\omega_L\tau}{B}(\mathbf{j}\times\mathbf{B}),$$

where $$\sigma = \frac{\tau e^2 n Z}{m}$$

is the electrical conductivity, ω_L is the Larmor frequency. This is an equation for the current density $\mathbf{j}$.

Ohm's law in the presence of magnetic field

The solution to this equation (Ohm's law) is

$$\mathbf{j} = \sigma_{\|}\mathbf{E}_{\|} + \sigma_{\perp}\mathbf{E}_{\perp} + \sigma_H \frac{1}{B}\mathbf{B}\times\mathbf{E}_{\perp},$$

where the electrical conductivity is anisotropic:

$\sigma_{\|} = \sigma$, along the magnetic field lines

$\sigma_{\perp} = \dfrac{\sigma}{1+\omega_L^2\tau^2}$, perpendicular to the magnetic field lines

$\sigma_H = \dfrac{\sigma\omega_L\tau}{1+\omega_L^2\tau^2}$. perpendicular to magnetic and transverse electric fields. Here ω_L is the Larmor frequency, τ is the characteristic collision time.

Exercise. Prove that this is a solution by substituting it in the equation. The last term in the Ohm's law sometimes is called the Hall effect.

Conductivity tensor

The Ohm's law can expressed in terms of a conductivity tensor. If we choose $\mathbf{B} = (0, 0, B)$ then

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix} = \begin{pmatrix} \sigma_\perp & -\sigma_H & 0 \\ \sigma_H & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix},$$

where $j_1, \ldots, E_1, \ldots$ are components of the vectors of the current density and electric field.

In the solar corona,

$$\omega\tau >> 1,$$

therefore,

$$\sigma_\perp << \sigma_H << \sigma.$$

That means that the electric current perpendicular to the magnetic field is relatively small.

Exercise. Estimate $\omega\tau$ for solar corona conditions, $n = 10^8 \text{ cm}^{-3}$, $T = 10^6$ K $B = 10$ G.

MHD Equations

Consider plasma in an electro-magnetic field.

The Maxwell equations are (CGS units):

MHD approximation

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t},$$

neglect displacement current because MHD processes are slow compared to the speed of light

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho_e,$$

neglect separation of electric charges of electrons and ions – plasma quasi-neutrality

$$\mathbf{j} = \sigma(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B})$$ - Ohm's law in a non-magnetized plasma

$(\omega_L \tau \ll 1)$ moving with velocity **v;** recall the electric field transformation in a moving coordinate system: $\boldsymbol{E}' = \boldsymbol{E} + \frac{1}{c}\left(\mathbf{v}\times\boldsymbol{B}\right)$

Equations of conservation of mass, momentum and energy

Then, we combine the Maxwell equations with the equations of conservation of mass, momentum and energy:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

$$\rho \frac{d\mathbf{v}}{dt} = \rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}\right) = -\nabla P + \frac{1}{c}\mathbf{j}\times\mathbf{B},$$

$$\rho T \frac{dS}{dt} = Q - L,$$

where ρ is the mass density, P is the gas pressure, $\mathbf{v}$ is the velocity, $S = c_V \log(P/\rho^{\gamma})$ is the specific entropy, Q is an energy input, L is the energy loss rate (radiative losses).

Adiabatic equation

The energy conservation equation (the second law of thermodynamics):

$$\rho T \frac{dS}{dt} = Q - L,$$

where $S = c_V \log(P/\rho^\gamma)$ is the specific entropy, Q is an energy input, L is the energy loss rate (radiative losses).

When the energy input and radiative losses are negligible, we get the adiabatic equation:

$$\rho T \frac{dS}{dt} = 0, \quad \text{or} \quad \frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0.$$

It means that in a moving fluid element the entropy is conserved: S=const, $\left(\frac{P}{\rho^\gamma}\right) =$ const.

Equations for the internal energy

The energy conservation equation can be rewritten in terms of the internal energy density: $E = \dfrac{P}{(\gamma - 1)\rho}$ (for the ideal gas):

$$\rho\left(\frac{dE}{dt} + P\frac{d}{dt}\left(\frac{1}{\rho}\right)\right) = Q - L,$$

where the second term in the left-hand side describes the work of external forces to change the specific volume $1/\rho$.

Using the mass conservation equation we get: $\rho\dfrac{dE}{dt} + P\nabla\mathbf{v} = Q - L.$

Magnetic Field Diffusion

Applying curl to the Ohm's equation:

$$\mathbf{E} = \frac{1}{\sigma}\mathbf{j} - \frac{1}{c}(\mathbf{v}\times\mathbf{B}),$$

$$\nabla\times\mathbf{E} = \frac{1}{\sigma}\nabla\times\mathbf{j} - \frac{1}{c}\nabla\times(\mathbf{v}\times\mathbf{B}),$$

and using the Maxwell equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$

we get an equation for magnetic field strength $\mathbf{B}$:

$$\frac{\partial\mathbf{B}}{\partial t} = -c\nabla\times\boldsymbol{E} = -c\left(\frac{1}{\sigma}\nabla\times\mathbf{j} - \frac{1}{c}\nabla\times(\mathbf{v}\times\mathbf{B})\right), \text{ or}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \frac{c^2}{4\pi}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{B}\right).$$

This is "the induction equation" - a central equation for solar MHD theories.

Magnetic Reynolds Number

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{c^2}{4\pi} \nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{B} \right).$$

advection of magnetic field   magnetic field diffusion

The first term in the right-hand side describes advection of magnetic field, the second term corresponds to magnetic field diffusion due to Joule dissipation.

The relative importance of these terms for a process of a characteristic scale L, velocity v is determined by the magnetic Reynolds number R_M or Re_M:

$$R_M = \frac{\frac{vB}{L}}{\frac{c^2}{4\pi\sigma}\frac{B}{L^2}} = \frac{4\pi\sigma L v}{c^2}.$$

For typical coronal conditions: $T = 10^6$ K, $\sigma = 10^{12}$ s^{-1}, $L = 10^8$ cm, $v = 10^5$ cm/s,

$R_M \sim 10^4 >> 1.$

Coefficient of magnetic diffusion

For uniform σ the last term can be simplified:

$$\nabla\times(\nabla\times\mathbf{B}) = \nabla(\nabla\cdot\mathbf{B}) - \nabla^2\mathbf{B} = -\nabla^2\mathbf{B}.$$

Then, if $\mathbf{v}=0$ we get a diffusion equation:

$$\frac{\partial\mathbf{B}}{\partial t} = D\nabla^2\mathbf{B},$$

where $D = \dfrac{c^2}{4\pi\sigma}$ is a diffusion coefficient for magnetic field.

Exercises:

1. Estimate the characteristic scale of dissipation of magnetic field in solar flares. The duration of solar flares is 10^3 sec.

$$L \sim \sqrt{\frac{c^2 t}{4\pi\sigma}} \sim 10^5\,\text{cm} = 1\text{km}.$$

2. This is smaller than the observed flare structure. What does that mean?

3. Estimate the decay time of sunspots ($L \sim 10^9$ cm, $T \sim 10^4$ K, $\sigma \sim 10^9$ s^{-1}).

$$t \sim \frac{4\pi\sigma L^2}{c^2} \sim 10^7 \text{ sec} \sim 4 \text{ months}.$$

4. This is longer than the observed lifetime of sunspots. Why?

Ideal MHD approximation

The MHD equations without energy input and dissipation are called the `ideal MHD' approximation:

$$\frac{d\rho}{dt} + \rho\nabla\mathbf{v} = 0$$

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla P + \frac{1}{4\pi}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}$$

$$\frac{d}{dt}\left(\frac{P}{\rho^{\gamma}}\right) = 0$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\mathbf{v}\times\boldsymbol{B})$$

The ideal MHD approximation is often used for modeling processes with high magnetic Reynolds number.

Linearized ideal MHD equations are used to describe MHD waves.
Consider small perturbations of velocity, density, pressure and magnetic field:
$\boldsymbol{v},\ \rho',\ P',\ \boldsymbol{b}$ in uniform stationary plasma with constant magnetic field:
velocity $\boldsymbol{v}_0 = 0,$ density $\rho_0,$ magnetic field $\boldsymbol{B}_0$.

Frozen Magnetic Flux Approximation (in-depth topic)

Consider a high-conductivity plasma, $R_M >> 1$, or $\sigma = \infty$ ('ideal plasma'). Then the equations for magnetic field are:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),$$

$$\nabla \cdot \mathbf{B} = 0.$$

Consider a 1D case: $\mathbf{B} = (0, B, 0)$, $\mathbf{v} = (v, 0, 0)$, - plasma motion across the magnetic field lines, with B and v depending only on x:

$$\frac{\partial B}{\partial t} = -\frac{\partial}{\partial x}(vB).$$

From this and the mass equation:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v),$$

we get

$$\frac{dB}{dt} + B\frac{\partial v}{\partial x} = 0,$$

$$\frac{d\rho}{dt} + \rho\frac{\partial v}{\partial x} = 0.$$

Then

$$\frac{dB}{dt} - \frac{B}{\rho}\frac{d\rho}{dt} = 0,$$

or

$$\frac{d\left(\frac{B}{\rho}\right)}{dt} = 0,$$ or $$\frac{B}{\rho} = \text{const.}$$

For a general 3D case:

$$\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right) = \left(\frac{\mathbf{B}}{\rho}\nabla\right)\mathbf{v}.$$

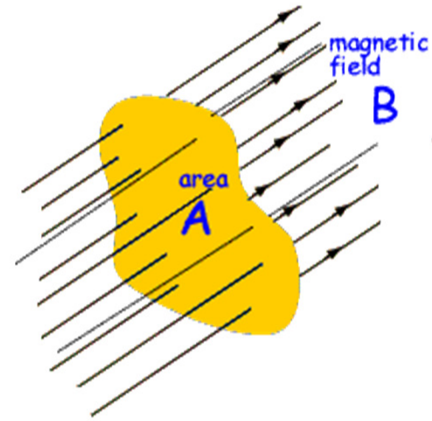
This equation shows that in ideal plasma magnetic field is coupled with plasma density. It follows the plasma motions.

Magnetic Flux Conservation (in-depth topic)

Consider magnetic flux Φ through a plasma area S restricted by a closed curve Γ:

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{s},$$

where $\mathbf{s}$ is a vector perpendicular to the area.
If we change the contour line of this plasma element then the total flux will change due the change of the magnetic field strength as follows from the MHD equations and due to the change of the area of this element:



$$\frac{d\Phi}{dt} = \frac{d\Phi'}{dt} + \frac{d\Phi''}{dt},$$

where

$$\frac{d\Phi'}{dt} = \iint_S \frac{\partial \mathbf{B}}{\partial t} d\mathbf{s}.$$

Now consider a small change of the area $d\mathbf{s}$ due to plasma motion with velocity $\mathbf{v}$ during time dt:

$$d\mathbf{s} = \mathbf{v}dt \times d\mathbf{l},$$

where $\mathbf{l}$ is the change of the length of contour Γ. Then the change of the magnetic flux:

$$d\Phi'' = \mathbf{B} \cdot d\mathbf{s} = \mathbf{B} \cdot \mathbf{v} \times d\mathbf{l}dt = -dt(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l},$$

where we used a vector-product relation. Then,

$$\frac{d\Phi''}{dt} = -\oint_\Gamma \mathbf{v} \times \mathbf{B} d\mathbf{l}.$$

Using Stokes' theorem to replace the contour integral with the surface integral we obtain for the total flux:

$$\frac{d\Phi}{dt} = \iint_S \frac{\partial \mathbf{B}}{\partial t} d\mathbf{s} - \oint_\Gamma \mathbf{v} \times \mathbf{B} d\mathbf{l}, \qquad \frac{d\Phi}{dt} = \iint_S \left[\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) \right] d\mathbf{s} = 0.$$

The right-hand side is equal zero because it satisfies the equation for magnetic field in an ideal plasma.

Thus, $\frac{d\Phi}{dt} = 0$. This is **the frozen flux theorem: the total magnetic flux through a plasma element does not change under deformations of this element, that is magnetic field moves with the plasma.**

Magnetic Forces

The Lorentz force, $\frac{1}{c}\mathbf{j}\times\mathbf{B}$, in the momentum equation can be expressed in terms of magnetic field:

$$\mathbf{f} = \frac{1}{4\pi}\nabla\mathbf{B}\times\mathbf{B}.$$

Using the standard vector formula: $\frac{1}{2}\nabla\mathbf{a}^2 = (\mathbf{a}\nabla)\mathbf{a} + \mathbf{a}\times\nabla\mathbf{a},$

$$\mathbf{f} = -\frac{1}{8\pi}\nabla B^2 + \frac{1}{4\pi}(\mathbf{B}\nabla)\mathbf{B} = -\nabla P_M + \frac{1}{4\pi}(\mathbf{B}\nabla)\mathbf{B}.$$

The first term is a gradient of magnetic pressure: $P_M = B^2/8\pi$;
the second term describes magnetic tension force.
The tension force is analogous to restoring force of rubber bands.

The magnetic forces can be written in a tensor form:

$$\mathbf{f} = -\frac{\partial}{\partial x_k}T_{ik},$$

where $T_{ik} = \frac{1}{4\pi}\left(\frac{1}{2}\delta_{ik}B^2 - B_i B_k\right)$ are Maxwell stresses. The magnetic forces are anisotropic.

MHD Waves

Magnetic forces can produce additional restoring force to small perturbations in a magnetized plasma and cause oscillations.

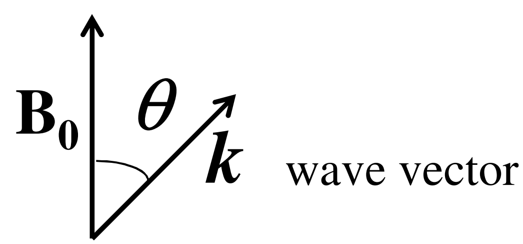
Consider a linearized system of the ideal MHD equations for perturbations ρ', velocity $\mathbf{v}$, and magnetic field, $\mathbf{b}$:

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \mathbf{v} = 0.$$

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla P' + \frac{1}{4\pi} \nabla \times \mathbf{b} \times \mathbf{B}_0$$

$$P' = \frac{\gamma P_0}{\rho_0} \rho' = c_S^2 \rho' \quad (c_S \text{ is the adiabatic sound speed})$$

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0)$$



Consider a periodic solution: $\rho', \mathbf{v}, \mathbf{b} \propto e^{i\mathbf{kr} - i\omega t}$

for $\mathbf{k} = (0, 0, k)$, $\mathbf{B}_0 = (0, B_0 \sin\theta, B_0 \cos\theta)$ - wave propagation along z-axis;

θ (the angle between $\mathbf{k}$ and $\mathbf{B}_0$).

If $B_0 = 0$ we obtain the dispersion relation for ordinary sound waves: $\omega^2 = c_S^2 k^2$.

The phase speed: $u = \omega / k = c_S$.

Alfven waves , fast and slow MHD waves

If $B_0 \neq 0$ then we have solutions (dispersion relations) of two types:

1) $$\omega^2 = k^2 \frac{B^2}{4\pi\rho} \cos^2\theta,$$

- **Alfven waves;** $v_A = B/\sqrt{4\pi\rho}$ **is the Alfven speed. These waves are incompressible** $\rho' = 0$**. Plasma moves perpendicular to magnetic field lines.** The phase speed $u = \omega/k$ is: $u_{Alfven} = \pm v_A \cos\theta$.

2) $$u^2 = \omega^2/k^2 = \frac{1}{2}(c_S^2 + v_A^2) \pm \frac{1}{2}\sqrt{v_A^4 + c_S^4 - 2v_A^2 c_S^2 \cos 2\theta},$$

where c_S is the sound speed.

The solution with "+" is called fast MHD wave, with "-" - slow MHD wave.

For $\theta = 0$ (propagation along field lines) and:

- $v_A > c_S$: $u_{fast} = v_A$, $u_{slow} = c_S$ - strong magnetic field
- $v_A < c_S$: $u_{fast} = c_S$, $u_{slow} = v_A$ - weak magnetic field

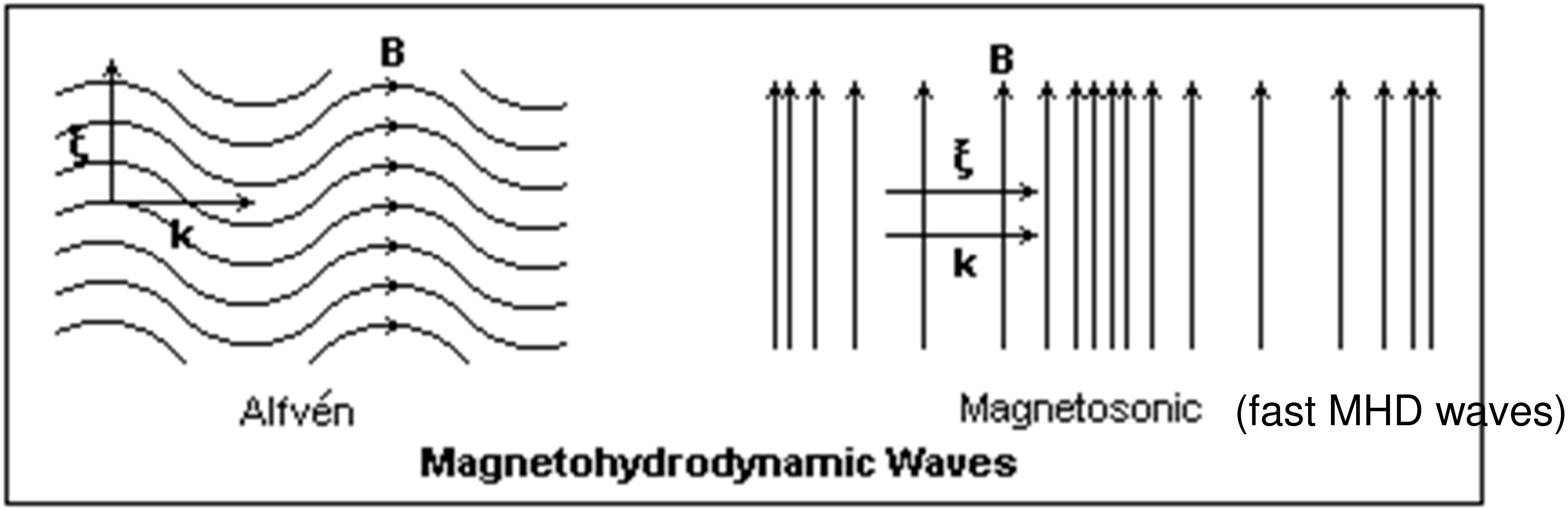
for $\theta = \pi/2$ (propagation perpendicular to the field lines):

- $u_{fast} = \pm\sqrt{v_A^2 + c_S^2}$, $u_{slow} = 0$

In the fast MHD waves the magnetic and pressure forces act together, in the slow MHD waves they act against each other.

In plasma of variable density (solar atmosphere) the waves can transform from one type to another.

Illustration of Alfven and magnetosonic (fast MHD) waves



- Alfven waves are incompressible. They transfer vorticity along the field lines. Plasma oscillates across the initial field lines.
- The fast MHD waves mostly travel across the magnetic field lines with a speed higher the speed of sound and the Alfven speed.
- The slow MHD waves mostly travel along the field line with a speed slower than the sound speed.

Animation of Alfven wave – magnetic field lines are frozen in plasma

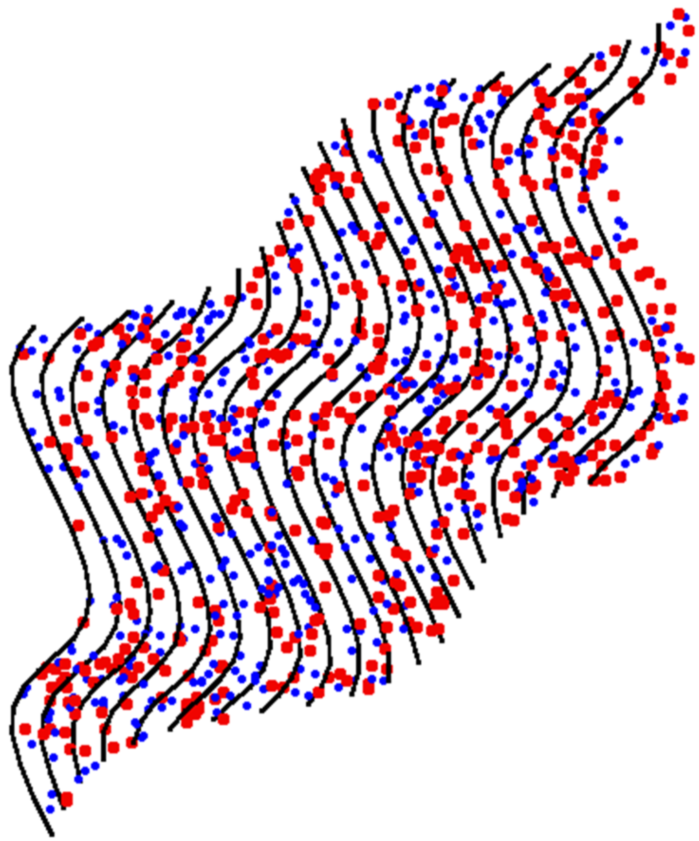
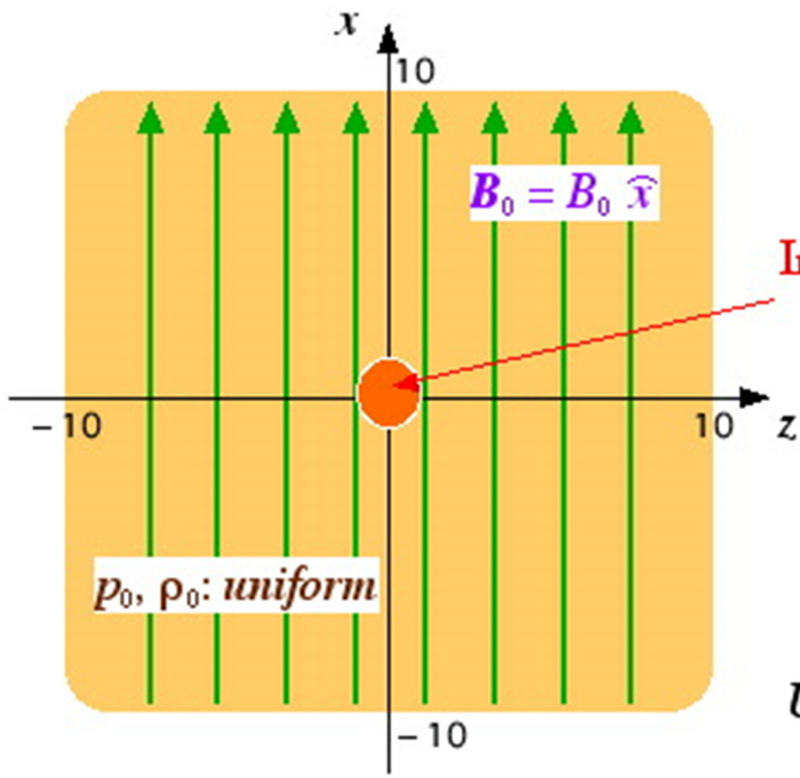


Illustration of the wave propagation and energy transport from a localized pressure perturbation (T. Magara)

Procedure of numerical simulation:

2-dimensional, adiabatic MHD simulation



Input initial perturbation:

$$p_0 \rightarrow p_0 + 0.1\,p_0$$

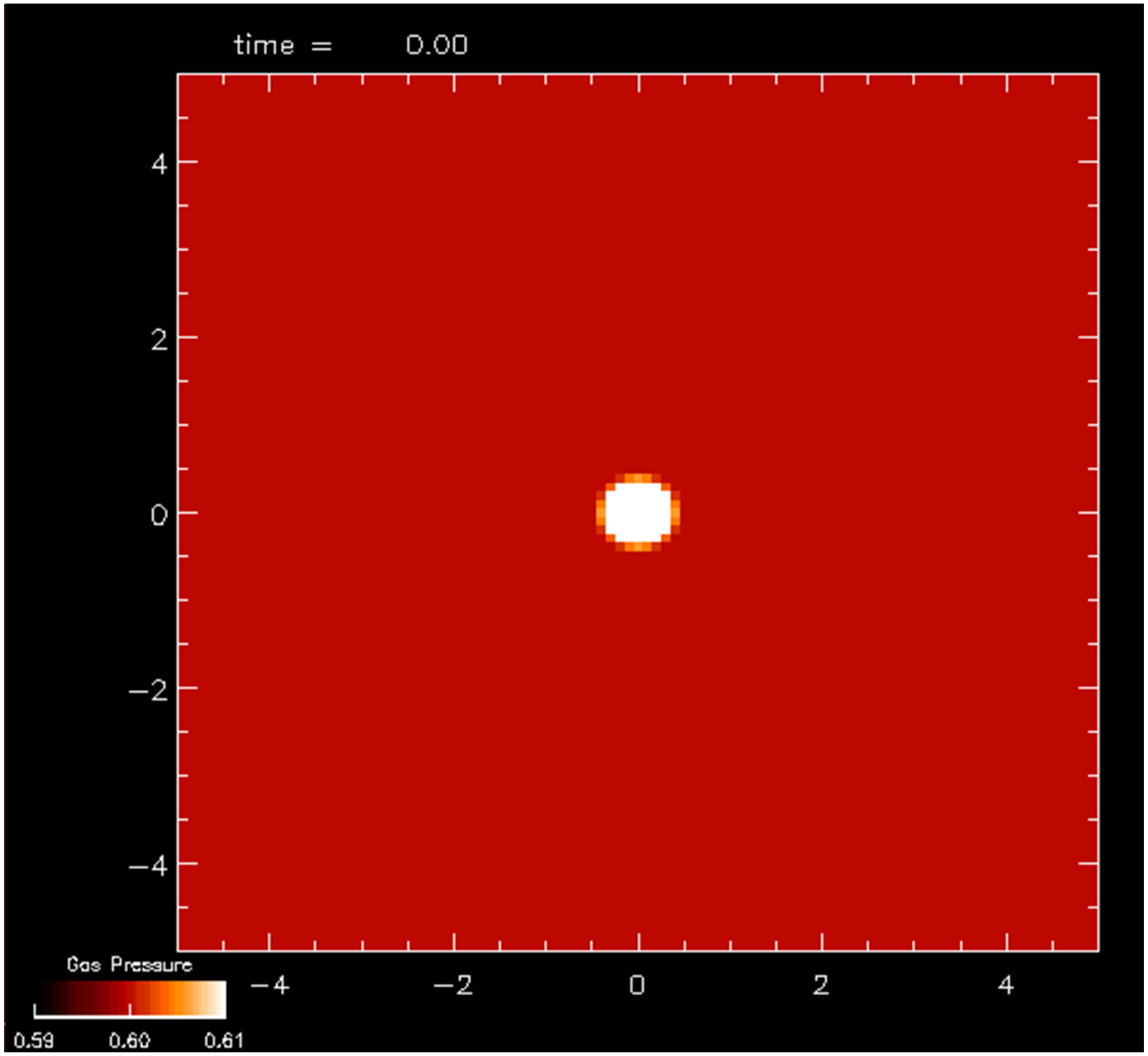
$$p(x, z) = p_0\left[0.05\left\{\cos(2\pi r) + 1\right\} + 1\right]$$

for $r \equiv \sqrt{x^2 + z^2} < 0.5$

Unit of velocity:

$$c_s = \sqrt{\gamma \frac{p_0}{\rho_0}} = 1$$

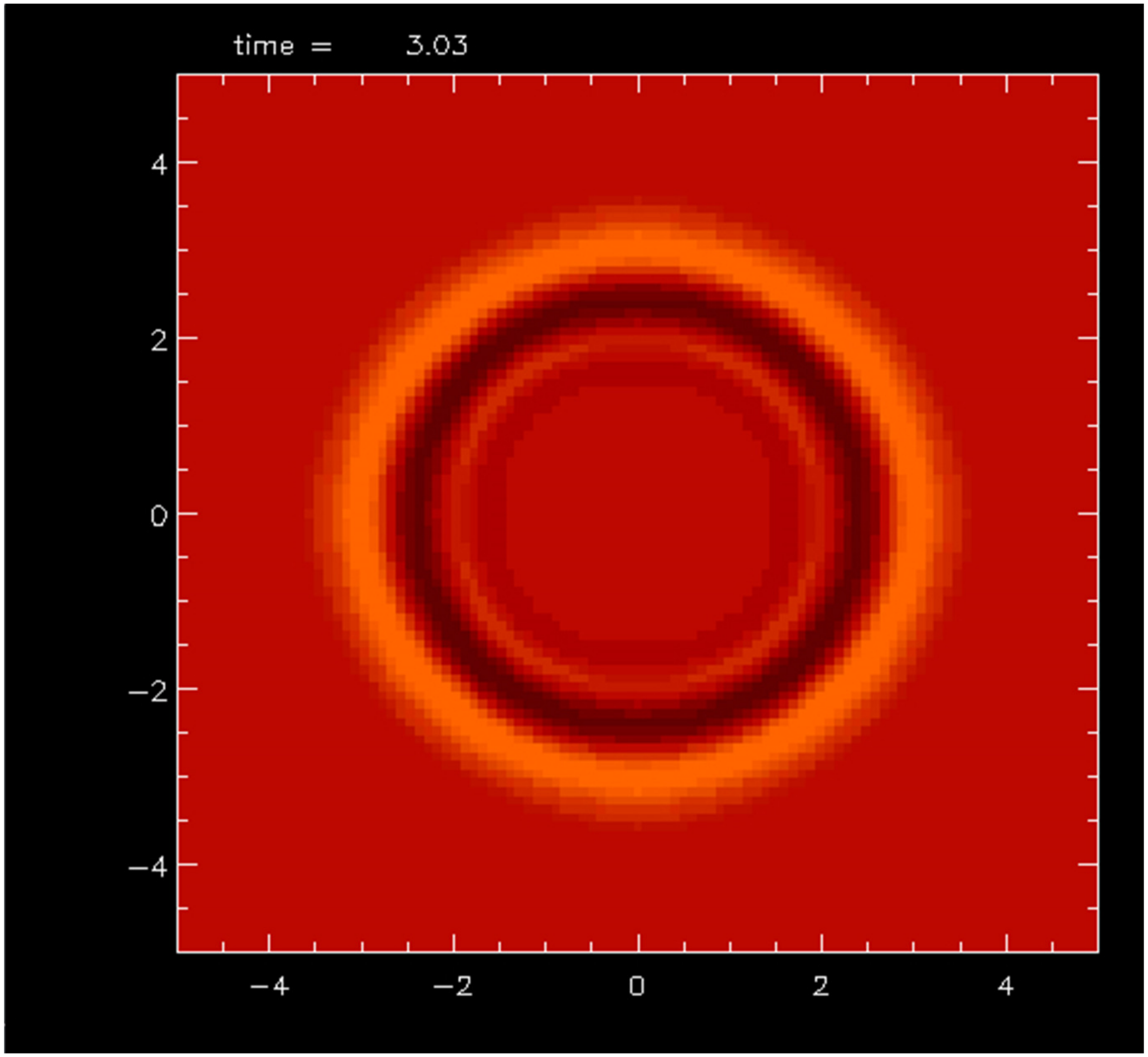
Sound wave (B_0=0)



Pressure perturbation

Without magnetic field the solution represents an isotropic sound wave.

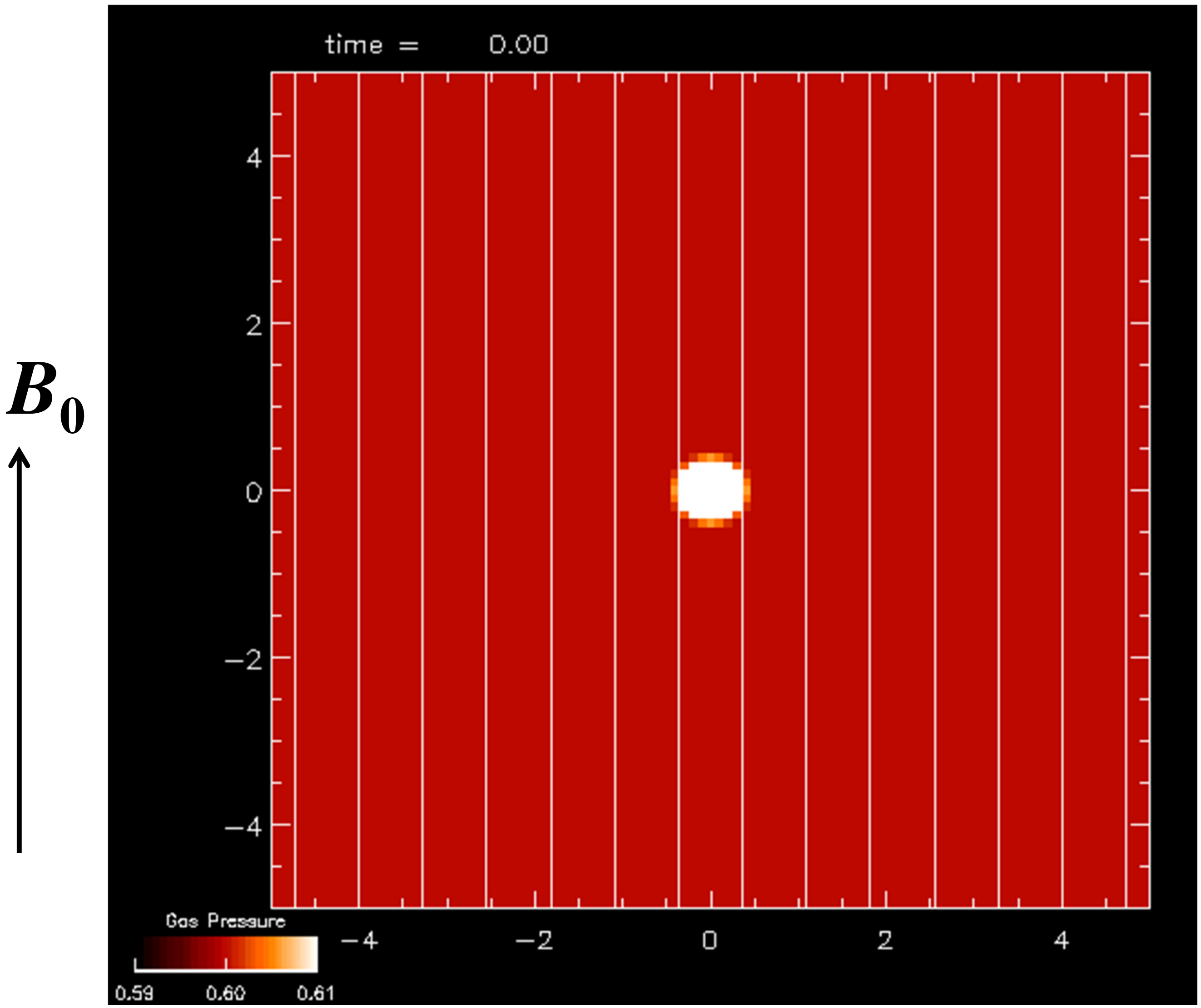
Sound wave (B_0=0)



Pressure perturbation

Without magnetic field the solution represents an isotropic sound wave.

Weak magnetic field ($c_S > v_A$)



Pressure perturbation

In the case of weak field:

$$c_S > c_A\text{, or}$$

$$\frac{\gamma P}{\rho} > \frac{B^2}{4\pi\rho}, \text{ or}$$

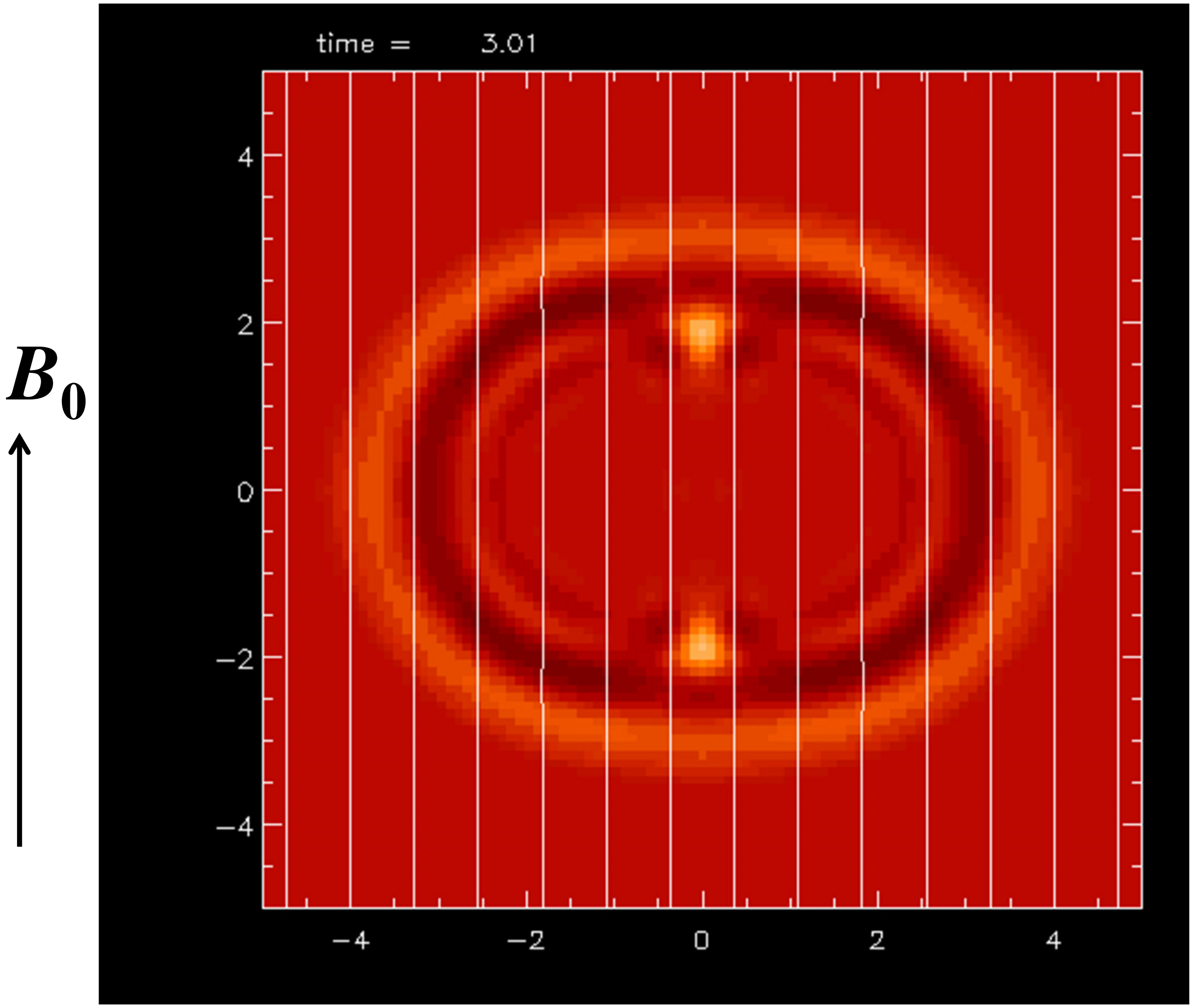
$$\frac{\gamma P}{B^2/4\pi} > 1, \text{ or}$$

$$P > P_M$$

the solution is a superposition of fast MHD waves (most pronounced across the field lines) and slow wave (mostly along the field lines) . This is the case of "high plasma beta":

$$\beta = \frac{P}{P_M} = \frac{8\pi P}{B^2}$$

Weak magnetic field ($c_S > v_A$)



Pressure perturbation

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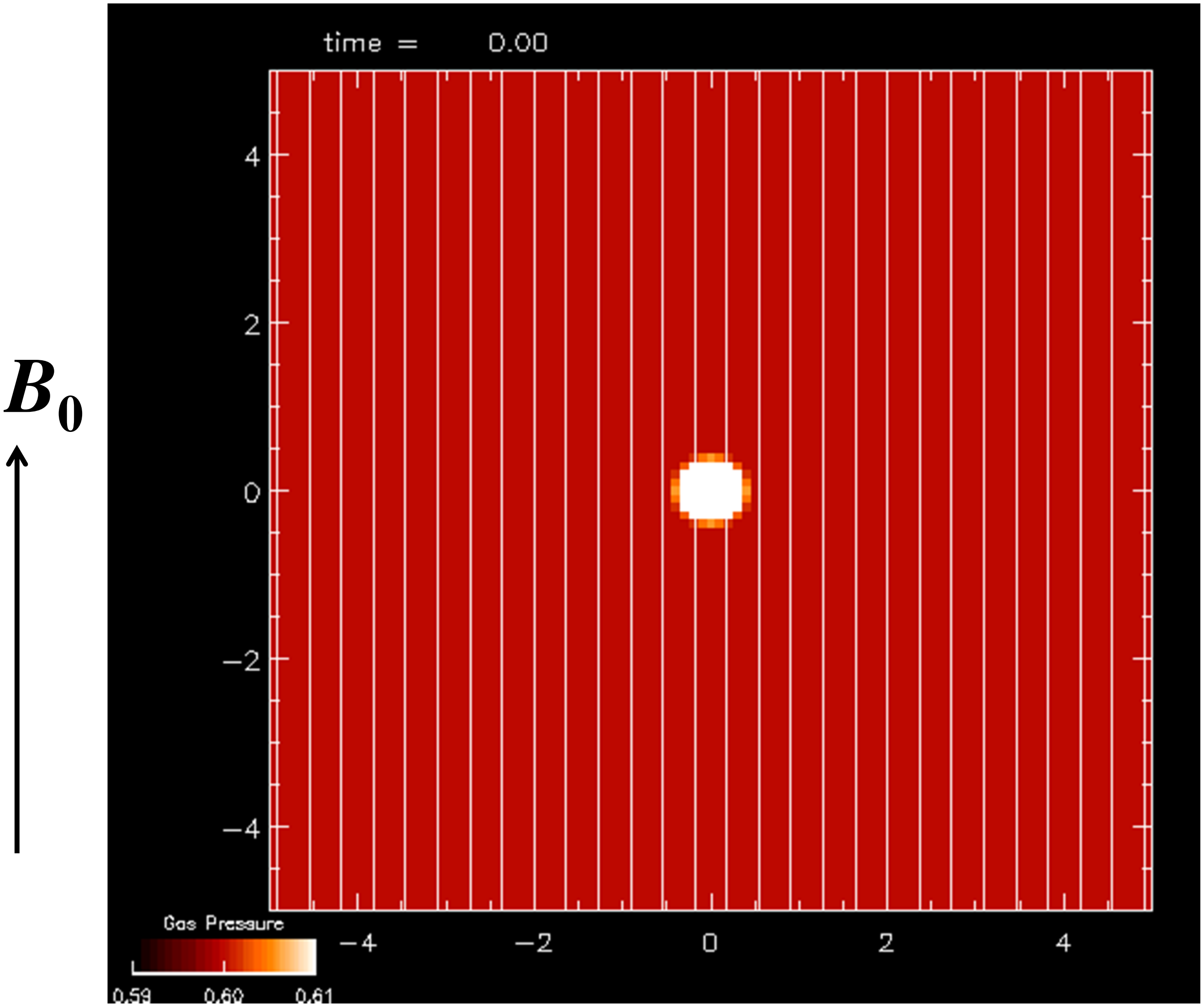
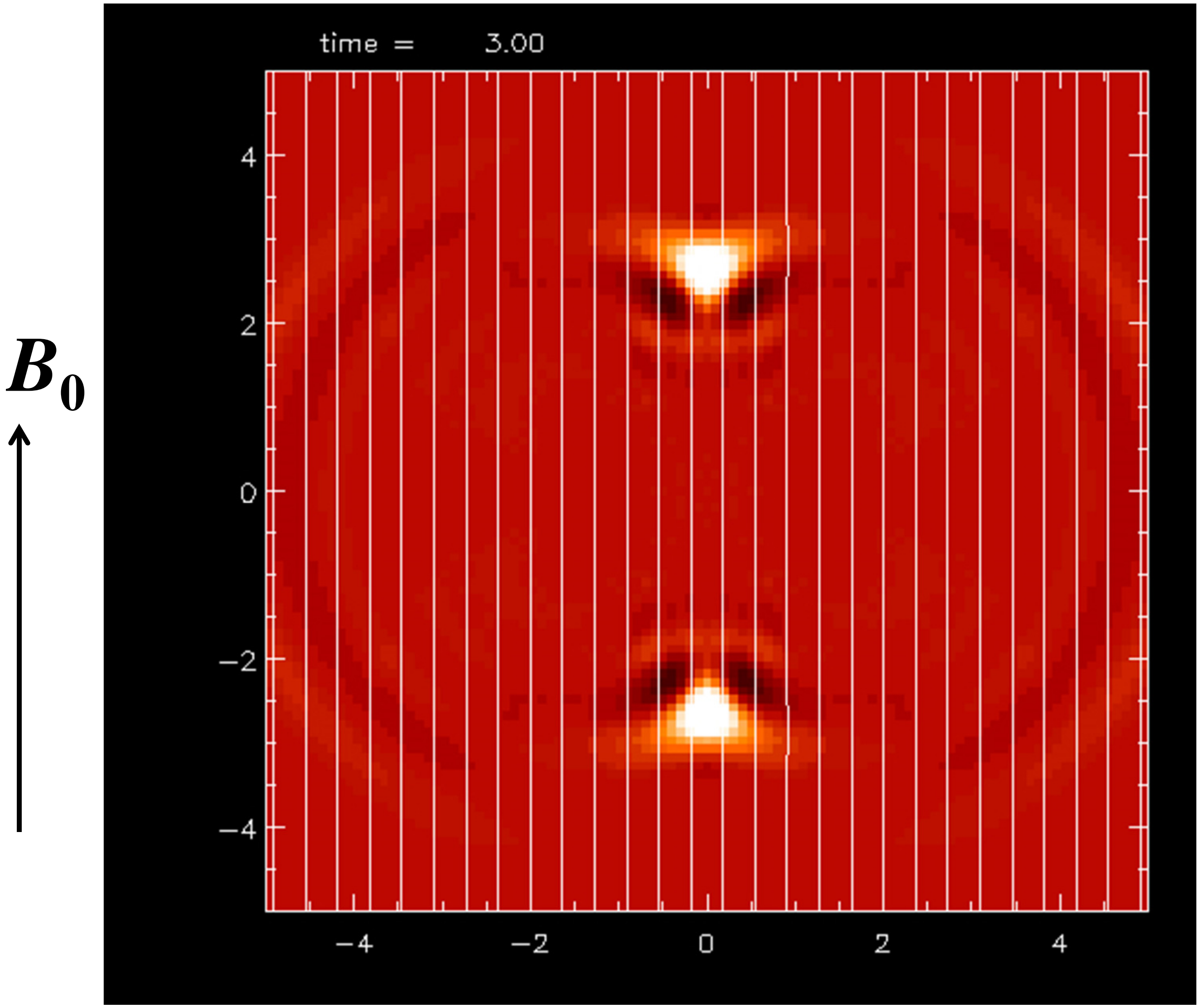
Strong magnetic field ($v_A > c_S$)



In this case ("low plasma beta") most of the energy is transported by the slow MHD waves along the magnetic field lines.

Pressure perturbation

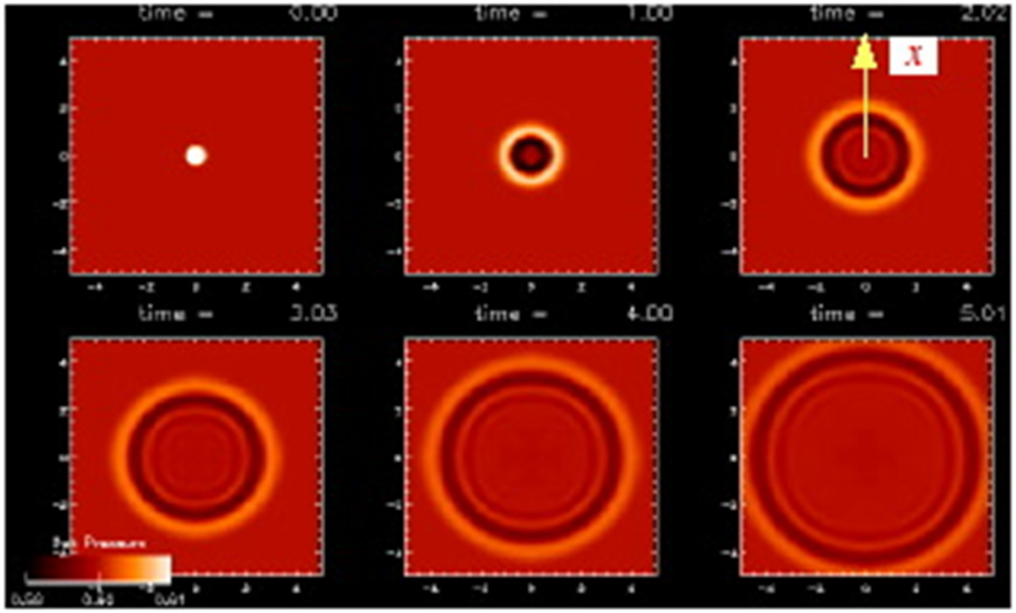
Strong magnetic field ($v_A > c_S$)



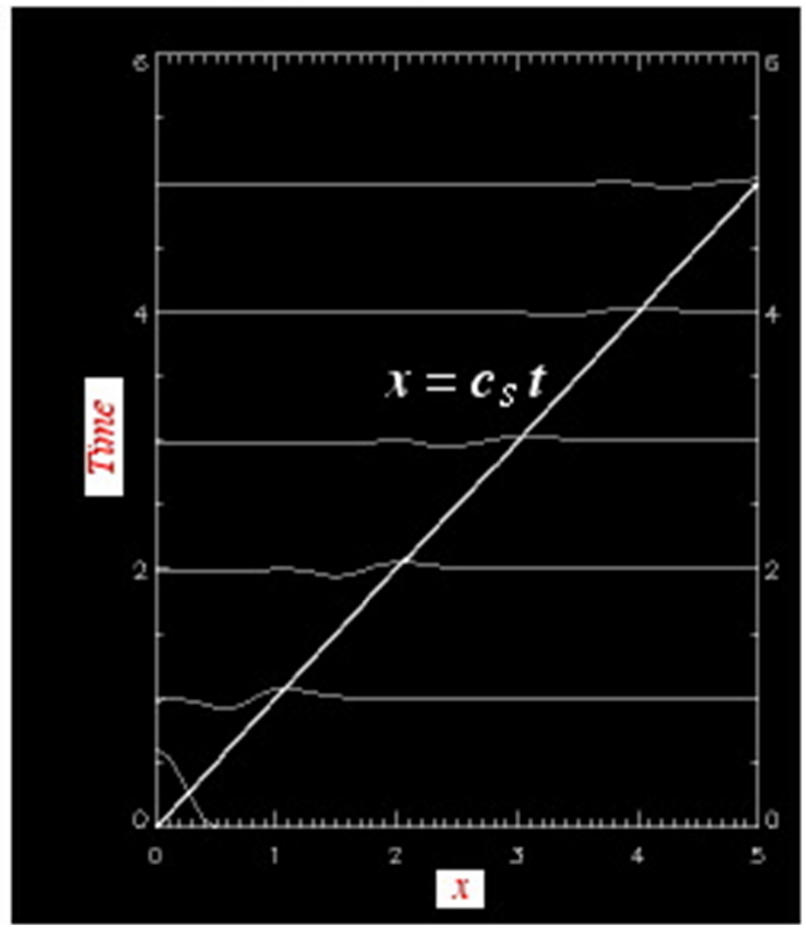
In this case ("low plasma beta") most of the energy is transported by the slow MHD waves along the magnetic field lines.

Pressure perturbation

Propagation of pure acoustic wave



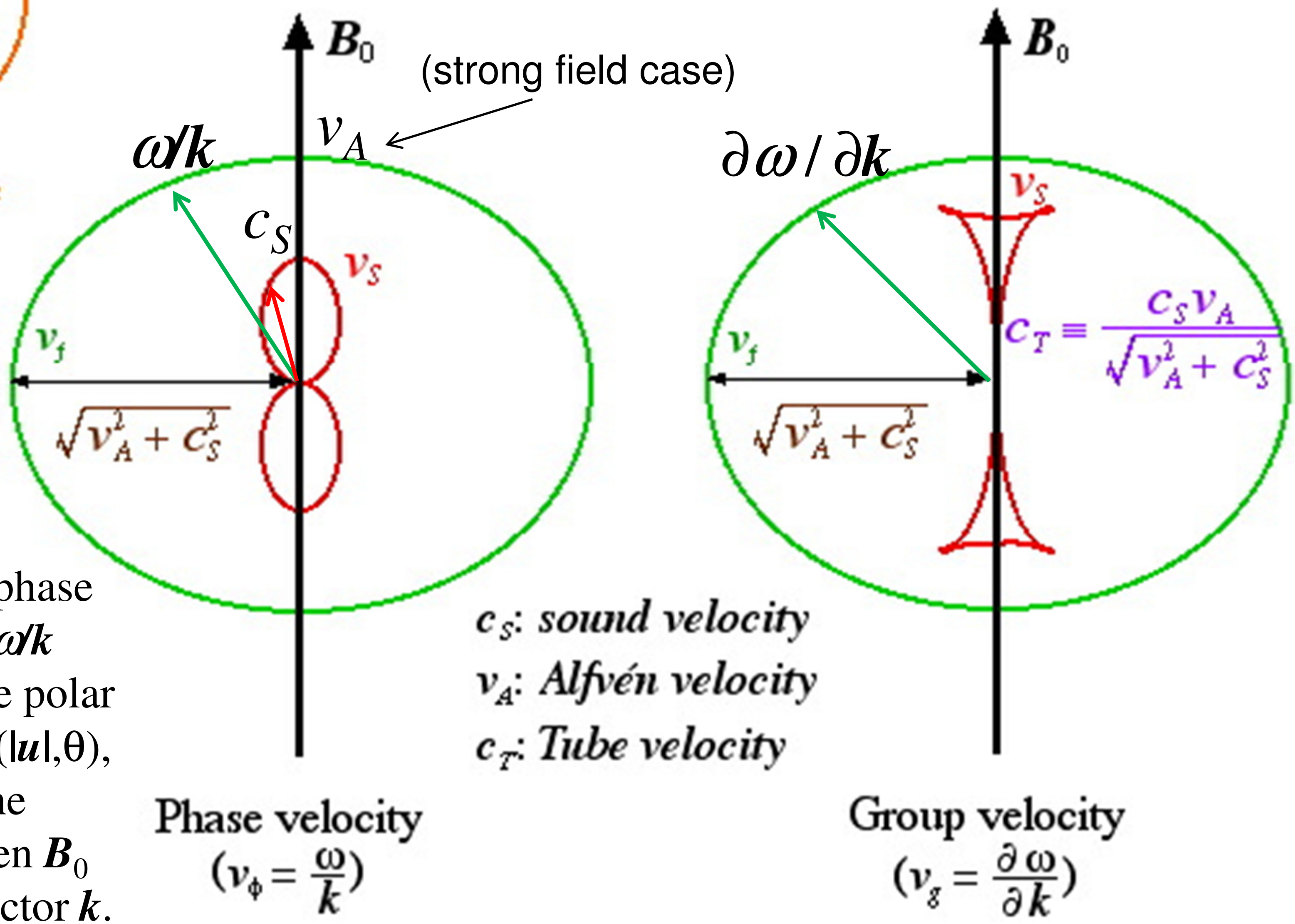
Propagation of sound wave



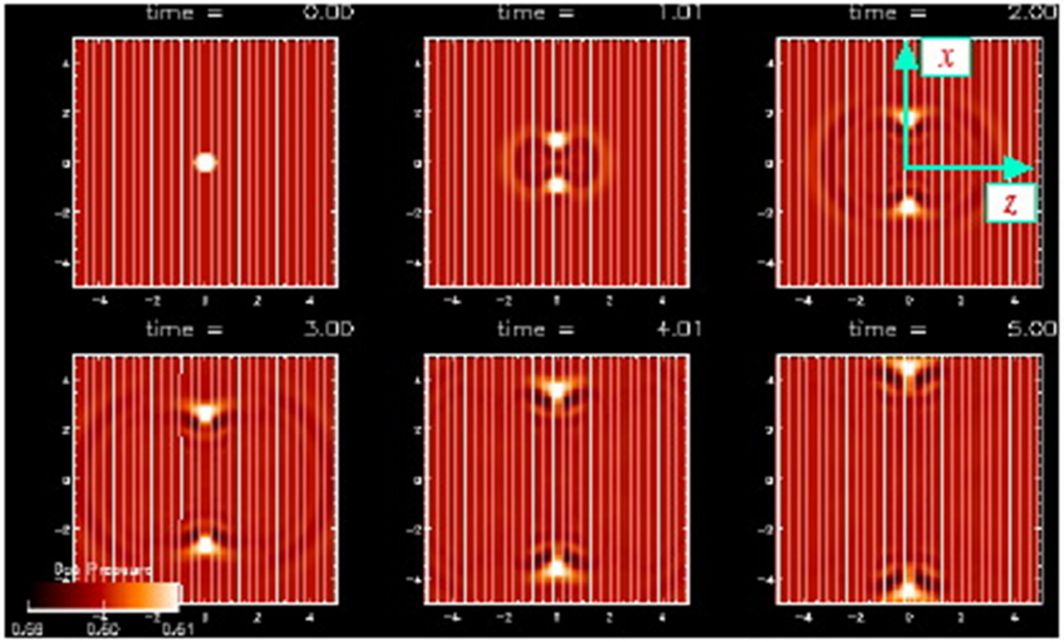
Time-distance diagram

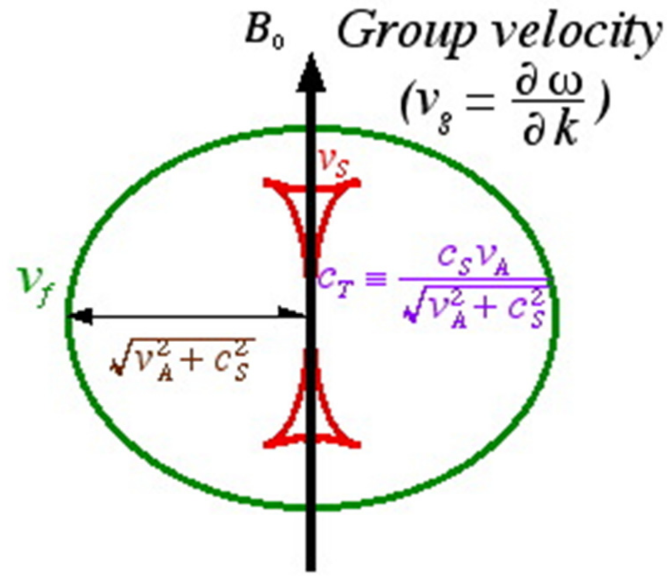
$$c_S = \sqrt{\gamma \frac{p_0}{\rho_0}} = 1$$

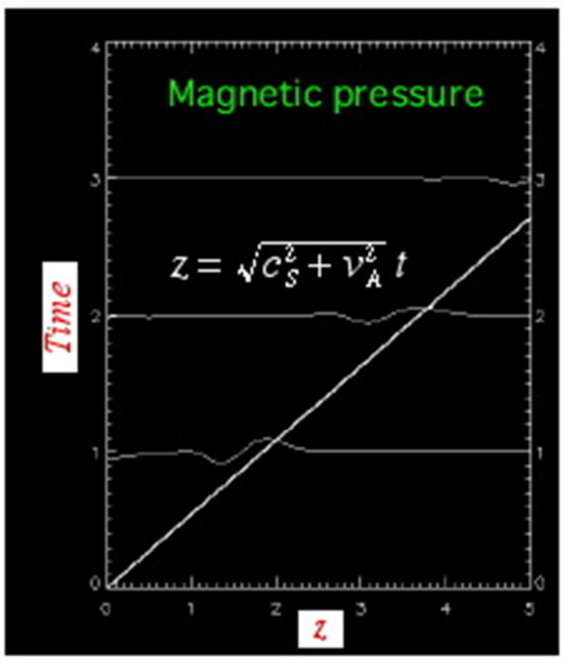
Explanation of MHD waves in terms of phase diagrams (Friedrich's diagrams)



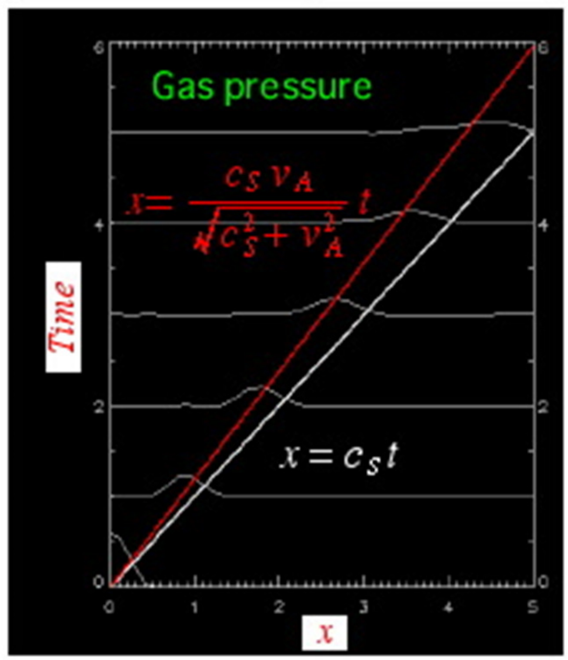
We plot the phase velocity, $\boldsymbol{u}=\omega/k$ (or v_ϕ), in the polar coordinates ($|\boldsymbol{u}|$,θ), where θ is the angle between $\boldsymbol{B}_0$ and wave vector $\boldsymbol{k}$.







Propagation of fast mode wave (perpendicular to B_0)



Propagation of slow mode & tube mode waves (along B_0)

$$c_S = \sqrt{\gamma \frac{p_0}{\rho_0}} = 1$$

$$v_A = \frac{B_0}{\sqrt{4\pi\rho_0}} = \sqrt{\frac{2}{\gamma\beta}}\, c_S = 1.55$$

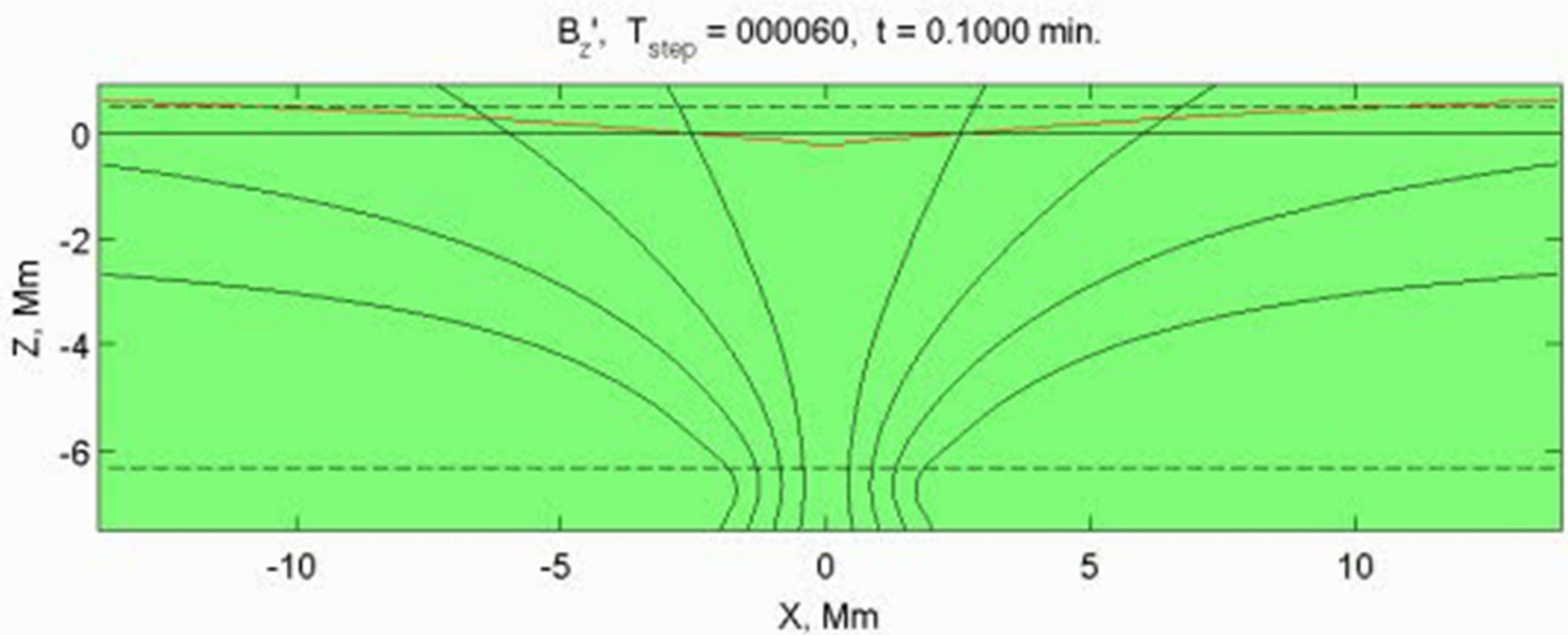
$$c_T = \frac{c_S v_A}{\sqrt{c_S^2 + v_A^2}} = 0.84$$

$$v_f = \sqrt{c_S^2 + v_A^2} = 1.84$$

Propagation of MHD waves: case $v_A > c_S$

Strong magnetic field transports wave energy preferentially along the magnetic field.

Numerical simulation of MHD wave propagation through a sunspot (K.Parchevsky)



Fast MHD waves travels across the magnetic field lines; slow MHD waves travels along the field lines into the deeper layers.