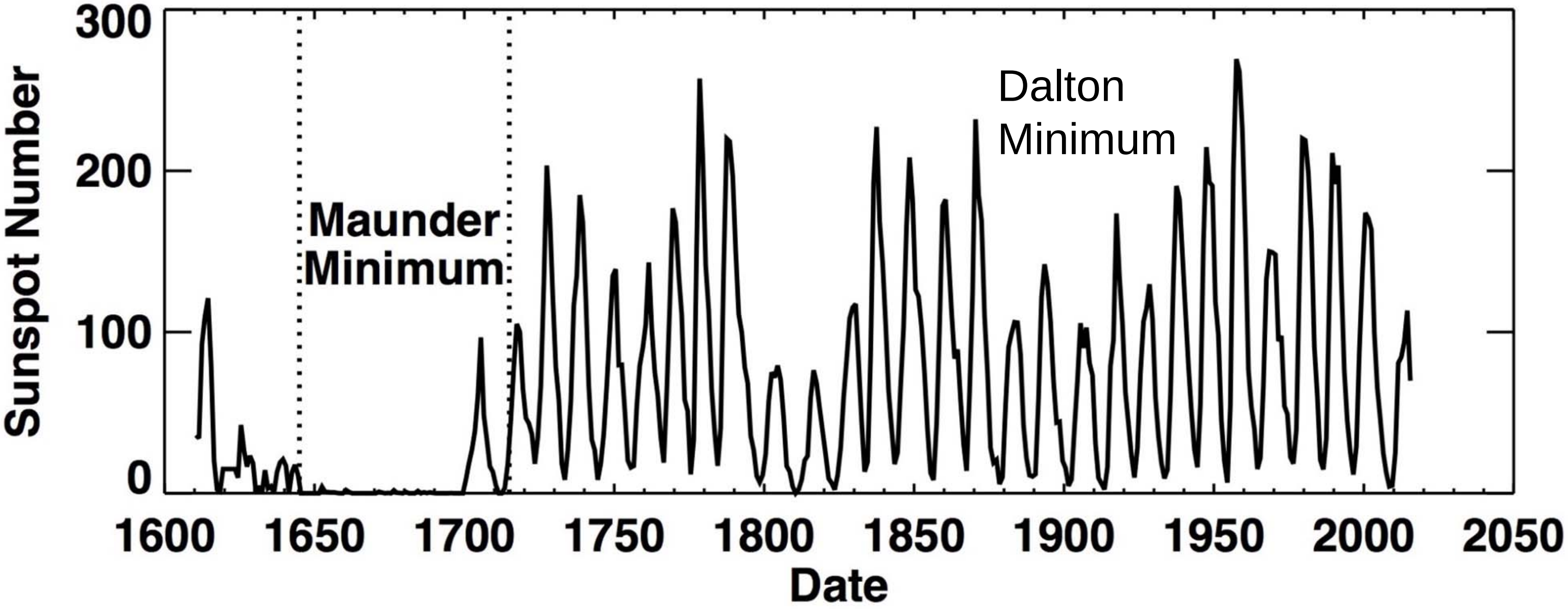
16. Solar Cycle and Dynamo Theory

Solar Dynamo

1. Solar Cycle
2. Babcock Model
3. Mean-Field Electrodynamics
4. Ω - and α -Effects
5. Kinematic Dynamo

Solar Cycle

Regular observations of sunspots have been made since the invention of the telescope, in the 17th century. In 1843 Schwabe was the first to notice that the number of sunspots visible on the Sun does not vary completely randomly but follows a cycle with a period of 11 year. The interval between the times of sunspot maximum varies somewhat, and may be as long as 15 years, or as short as 8 years.



The sunspot record. The low level of activity from 1645-1715 (the Maunder minimum) was not an observational problem.

Sunspot numbers

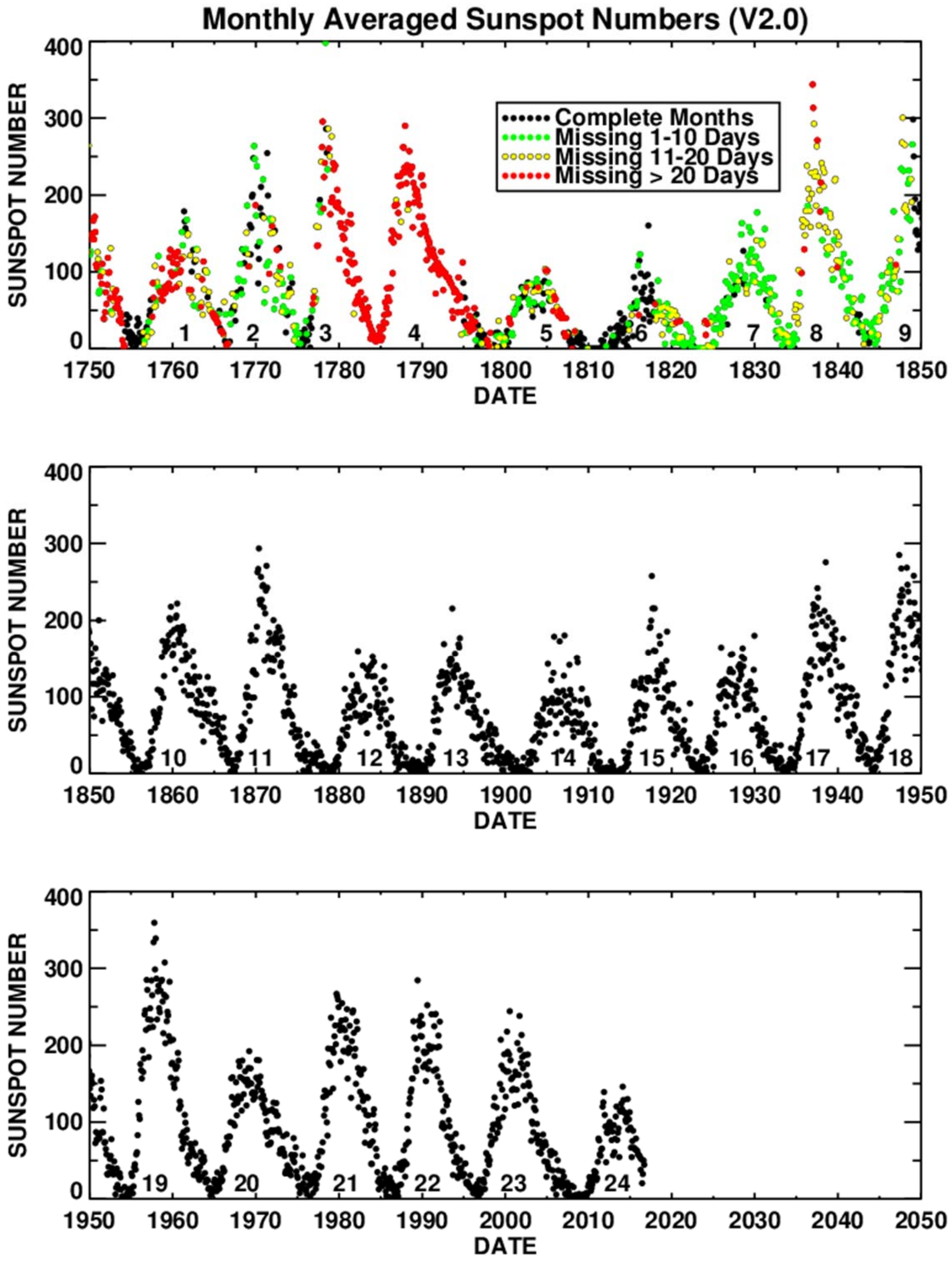
There are two official sunspot numbers in common use. The first, the daily "*Boulder Sunspot Number*," is computed by the NOAA Space Environment Center using a formula devised by Rudolph Wolf in 1848: **R=k (10g+s), where R is the sunspot number; g is the number of sunspot groups on the solar disk; s is the total number of individual spots in all the groups; and k is a variable scaling factor** (usually <1) that accounts for observing conditions and the type of telescope (binoculars, space telescopes, etc.). Scientists combine data from lots of observatories -- each with its own k factor -- to arrive at a daily value.

The Boulder number (reported daily on SpaceWeather.com) is usually about 25% higher than the second official index, the "***International Sunspot Number***," published daily by the Sunspot Index Data Center in Belgium. Both the Boulder and the International numbers are calculated from the same basic formula, but they incorporate data from different observatories.

As a rule of thumb, if you divide either of the official sunspot numbers by 15, you'll get the approximate number of individual sunspots visible on the solar disk if you look at the Sun by projecting its image on a paper plate with a small telescope.

Numbering of solar cycles starts from 1755.
The current cycle is number 24.

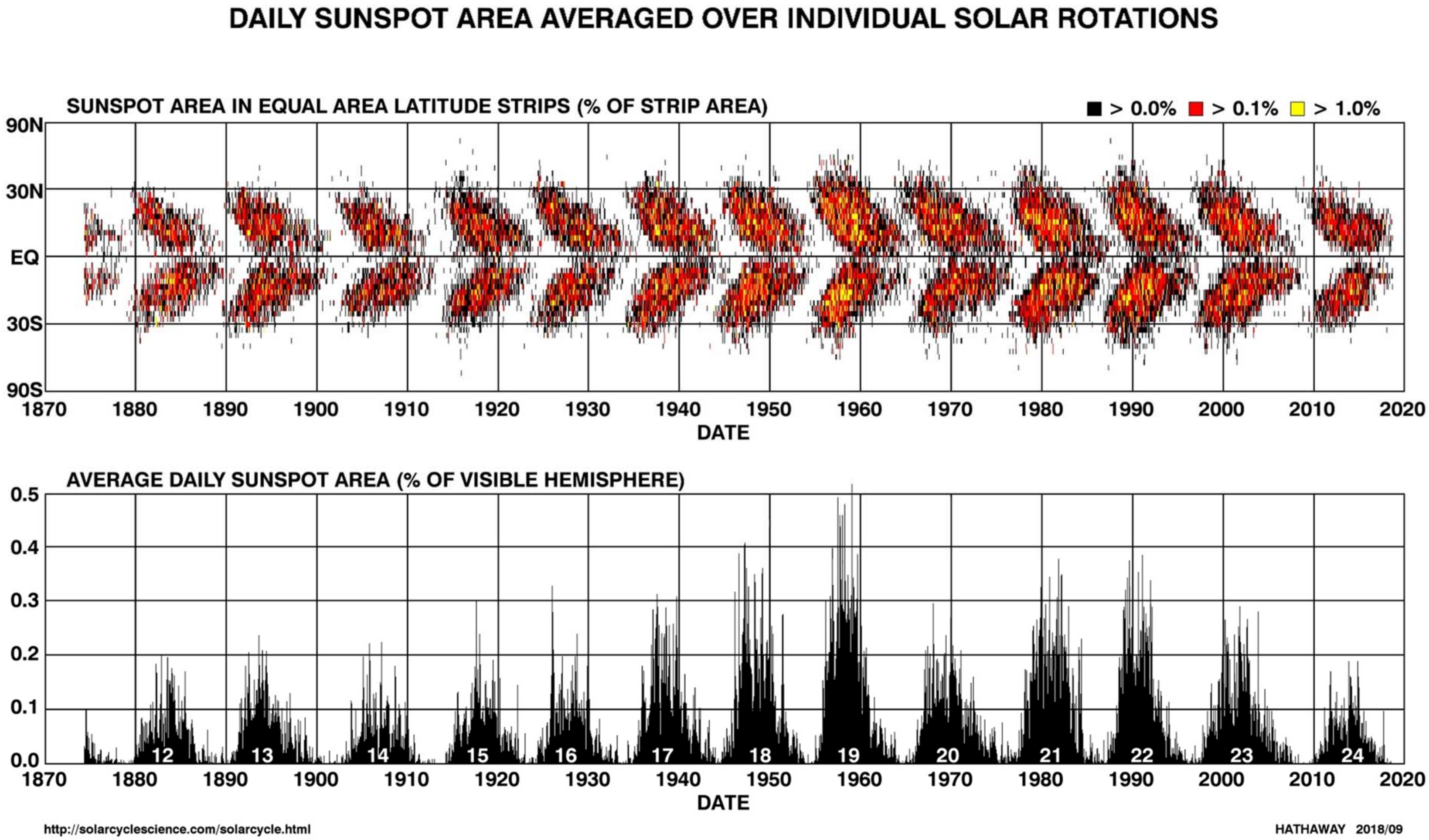
Waldmeier’s effects:
1) stronger cycles have shorter rise time;
2) weaker cycles are longer.



HATHAWAY SolarCycleScience.com 2018/06

The butterfly diagram

Carrington (1858) found that sunspots tend to appear at lower and lower latitude as the solar cycle progresses.



The butterfly diagram: the sunspot areas vs latitude and time. In the beginning of the cycle, sunspots appear at mid latitudes. As the cycle progresses, they are found close and closer to the equator.

Hale’s Polarity Law

Hale’s polarity law: sunspots appear in bipolar pairs, the leading spots have opposite polarity in the Northern and Southern hemispheres, and the order of these polarities is reverse from one solar cycle to the next cycle.

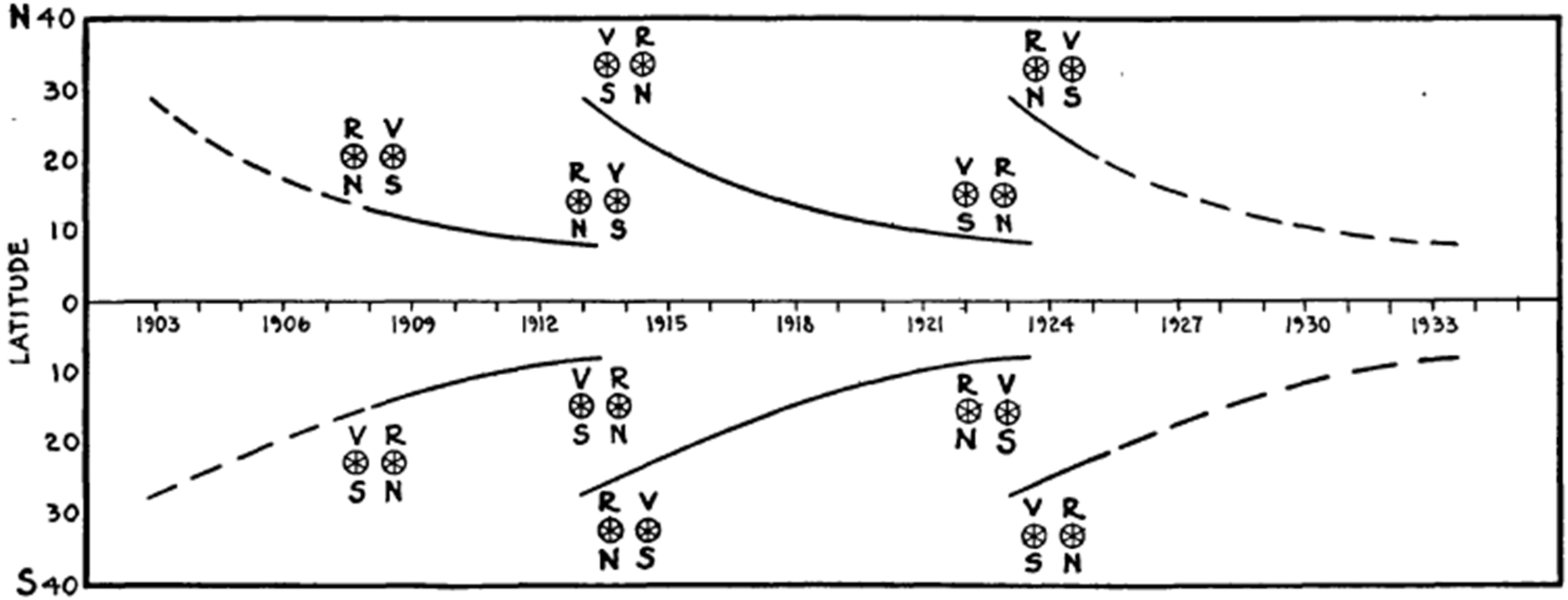


FIG. 18.—The law of sun-spot polarity. The curves represent the approximate variation in mean latitude and the corresponding magnetic polarities of spot groups observed at Mount Wilson from June 1908 to January 1925. The preceding spot is shown on the right.

During solar minima two consecutive cycles may overlap

Hale also found that during solar minima the magnetic fields of the new and old cycles may appear at the same time but at different latitudes.

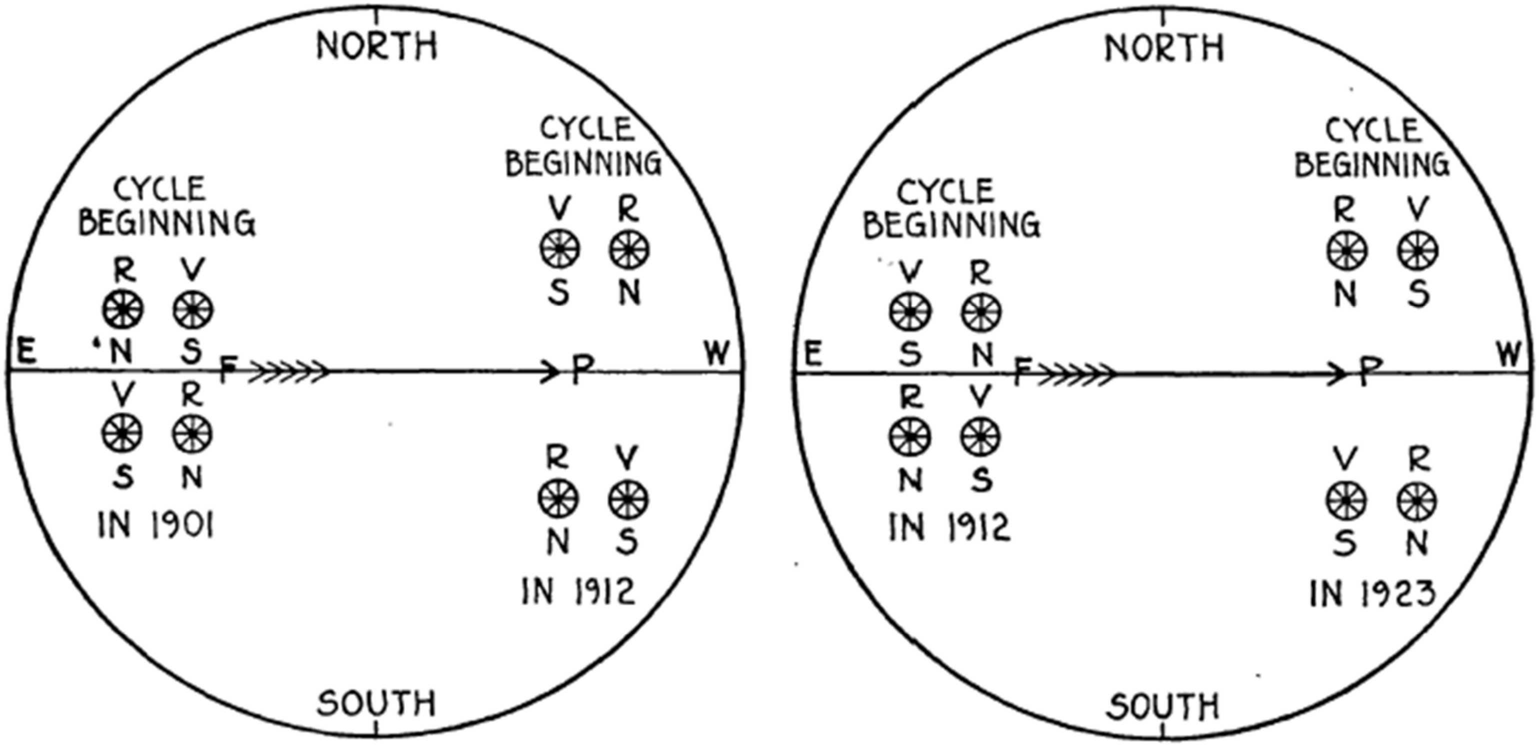
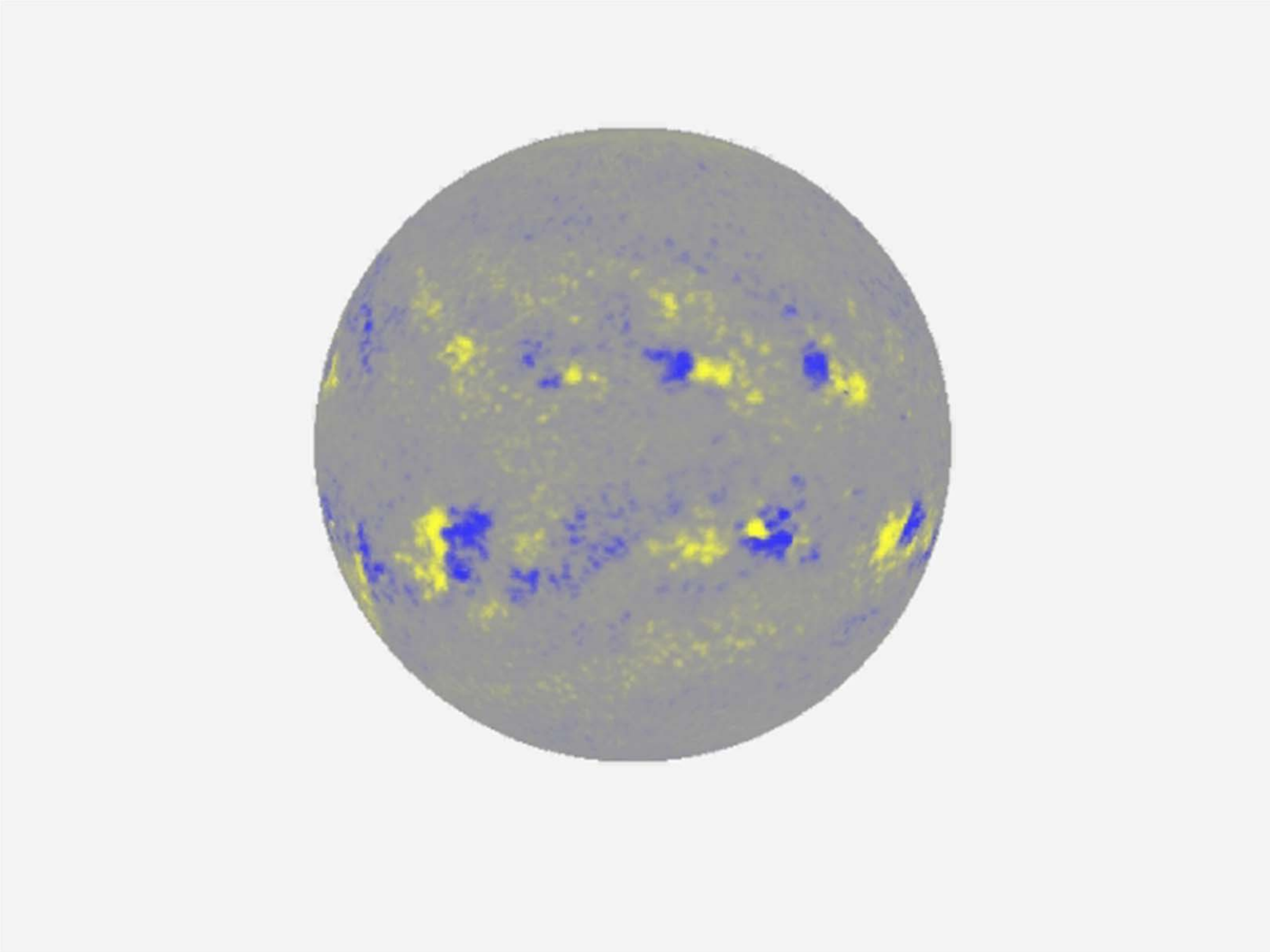


FIG. 19.—Sun-spot zones during the minimum of solar activity. Two zones in each hemisphere, in which the spots are of opposite magnetic polarity, exist for about two years at the time of each sun-spot minimum.

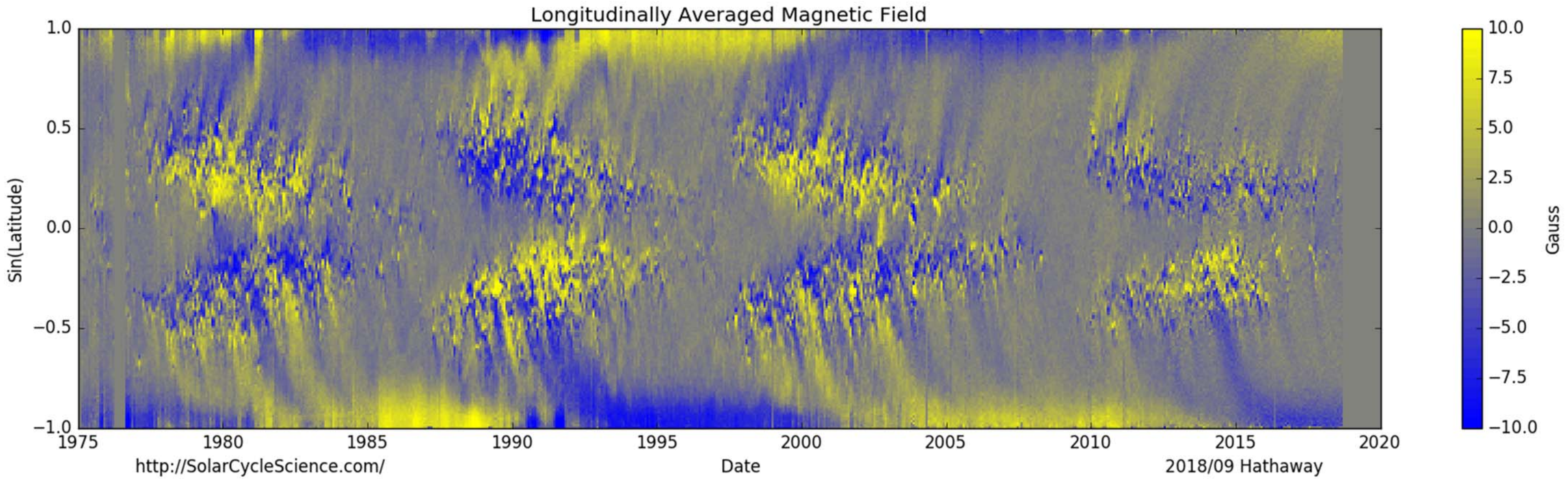
Evolution of the Sun's magnetic field over 30-years

(D. Hathaway)

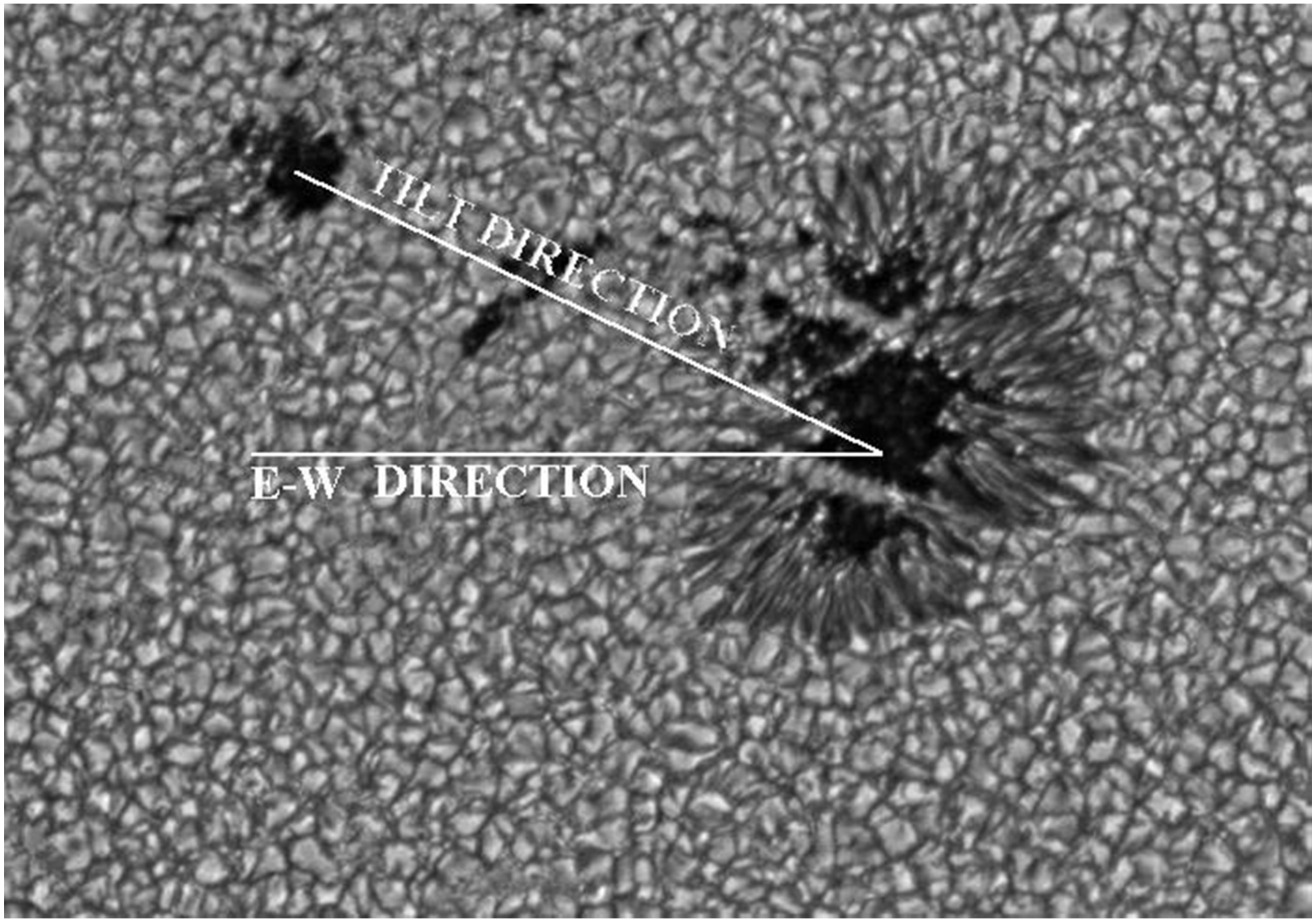


The magnetic butterfly diagram.

Axisymmetrical component of the line-of-sight magnetic field averaged over a full solar rotation as a function of time. Yellow and blue colors shows positive and negative magnetic fields.



Joy’s law: the leading spot is closer to equator than the following one



A successful model for the solar dynamo must explain several observations:

1. The 11-year period of the sunspot cycle,
2. The equator-ward drift of the active latitude as seen in the butterfly diagram
3. The Hale’s polarity law and the 22-year magnetic cycle,
4. The Joy’s law for the observed tilt of sunspot groups
5. The reversal of the polar magnetic fields near the time of cycle maximum as seen in the magnetic butterfly diagram.
6. The Waldmeier’s effect for the inverse relationship between the cycle amplitude and the rise time.

Mean-Field Electrodynamics

Consider the MHD induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c^2}{4\pi\sigma} \nabla^2 \mathbf{B} =$$

$$\equiv \nabla \times [\mathbf{v} \times \mathbf{B} - \frac{c^2}{4\pi\sigma} \nabla \times \mathbf{B}].$$

We seek a solution to the dynamo problem in terms of a mean magnetic field:

$$\mathbf{B} = <\mathbf{B}> + \mathbf{b},$$

where $\mathbf{b}$ is a fluctuating part of $\mathbf{B}$: $<\mathbf{b}> = 0$.
Similarly, we consider a global and fluctuating motions:

$$\mathbf{v} = <\mathbf{v}> + \mathbf{u},$$

where $<\mathbf{u}> = 0$. Then, we separate the large-scale and fluctuating parts of the induction equation:

$$\frac{\partial (<\mathbf{B}> + \mathbf{b})}{\partial t} = \nabla \times [(<\mathbf{v}> + \mathbf{u}) \times (<\mathbf{B}> + \mathbf{b}) - \frac{c^2}{4\pi\sigma} \nabla \times (<\mathbf{B}> + \mathbf{b})].$$

The mean part is: $$\frac{\partial <\mathbf{B}>}{\partial t} = \nabla \left[<\mathbf{v}> \times <\mathbf{B}> + <\mathbf{u} \times \mathbf{b}> - \frac{c^2}{4\pi\sigma} \nabla \times <\mathbf{B}> \right].$$

extra term due correlated turbulent fluctuations

Electromotive Force

The term $E = < \mathbf{u} \times \mathbf{b} >$

in the mean-field equation represents a mean electric field generated by fluctuating magnetic and velocity fields ("electromotive force", emf).
If it is known we can solve the mean-field equation for $< \mathbf{B} >$.

In principle, E must be calculated in terms of $< \mathbf{B} >$ using the equation for the fluctuating part, $\mathbf{b}$. In general, this is difficult because requires the knowledge of the turbulent fluctuations.

However, there is a linear relation between $\mathbf{b}$ and $< \mathbf{B} >$, and hence between E and $< \mathbf{B} >$. We write this relation as an expansion:

$$E = \alpha < \mathbf{B} > - \beta \nabla \times < \mathbf{B} > + \ldots$$

For almost isotropic turbulence:

$$\alpha \simeq \frac{1}{3} < \mathbf{u} \cdot \nabla \times \mathbf{u} > \tau,$$

$$\beta \simeq \frac{1}{3} < \mathbf{u} \cdot \mathbf{u} > \tau,$$

where τ is a characteristic correlation time of turbulent fluctuations.

The α-effect and kinetic helicity

The first term of the electromotive force

$$E = \alpha < \mathbf{B} > - \beta \nabla \times < \mathbf{B} > + \ldots$$

where $$\alpha \simeq \frac{1}{3} < \mathbf{u} \cdot \nabla \times \mathbf{u} > \tau, \;\; \beta \simeq \frac{1}{3} < \mathbf{u} \cdot \mathbf{u} > \tau,$$

is called α **-effect**, and $< \mathbf{u} \cdot \nabla \times \mathbf{u} >$ is called '**kinetic helicity'**.
The coefficient β is called 'turbulent diffusivity'.

Then, the mean-field equation is:

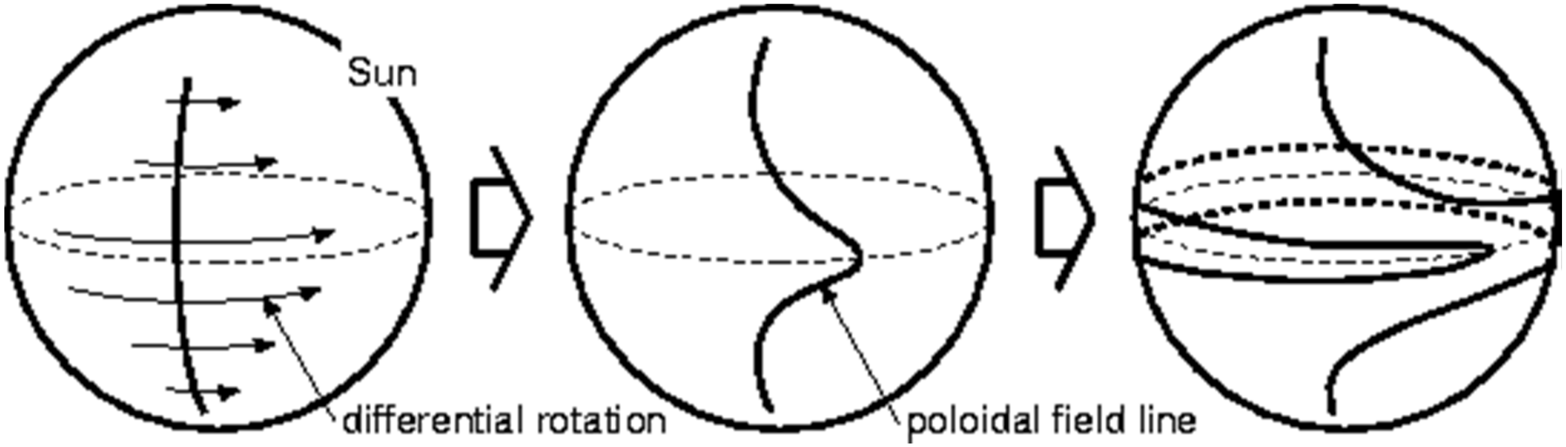
$$\frac{\partial < \mathbf{B} >}{\partial t} = \nabla \times \left(< \mathbf{v} > \times < \mathbf{B} > + \alpha < \mathbf{B} > - (\eta + \beta) \nabla \times < \mathbf{B} > \right),$$

where $\eta = \frac{c^2}{4\pi\sigma}$ is magnetic diffusivity. The sum $\eta + \beta \equiv \eta_T$ is the total magnetic diffusivity. Since $\eta \ll \beta$ the turbulent diffusivity dominates.
The α -term generates magnetic field by providing the dynamo effect.

Reference: Equation for α and β are derived in A.R. Choudhuri, The Physics of Fluids and Plasmas, Cambridge Univ. Press, 1998.

Ω - and α -Effects

Consider an initially poloidal (in the r and θ directions, in spherical polar coordinates) fossil field subject to a differential azimuthal flow within the Sun. The field is stretched by the flow in the toroidal direction (the azimuthal direction) a process called the Ω effect.



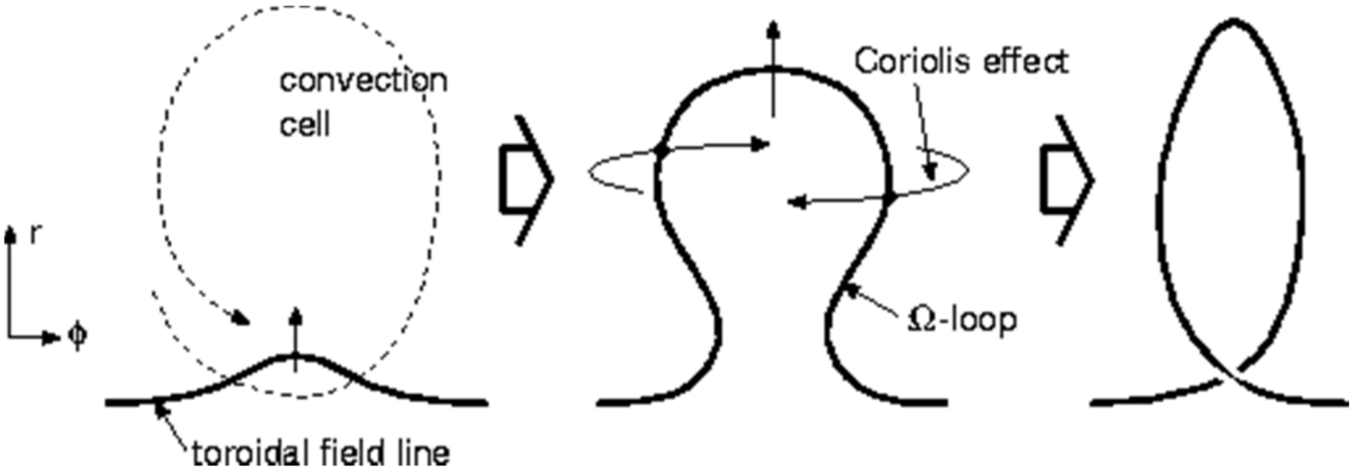
Poloidal field

Toroidal field

Ω effect

The origin of the α-effect

To close the dynamo cycle, it is necessary to have a scheme whereby a poloidal field is reproduced from the stretched toroidal field bands produced by the Ω effect. This is argued to occur through cyclonic convection. A toroidal field line caught in a convection cell will be pulled into a loop.



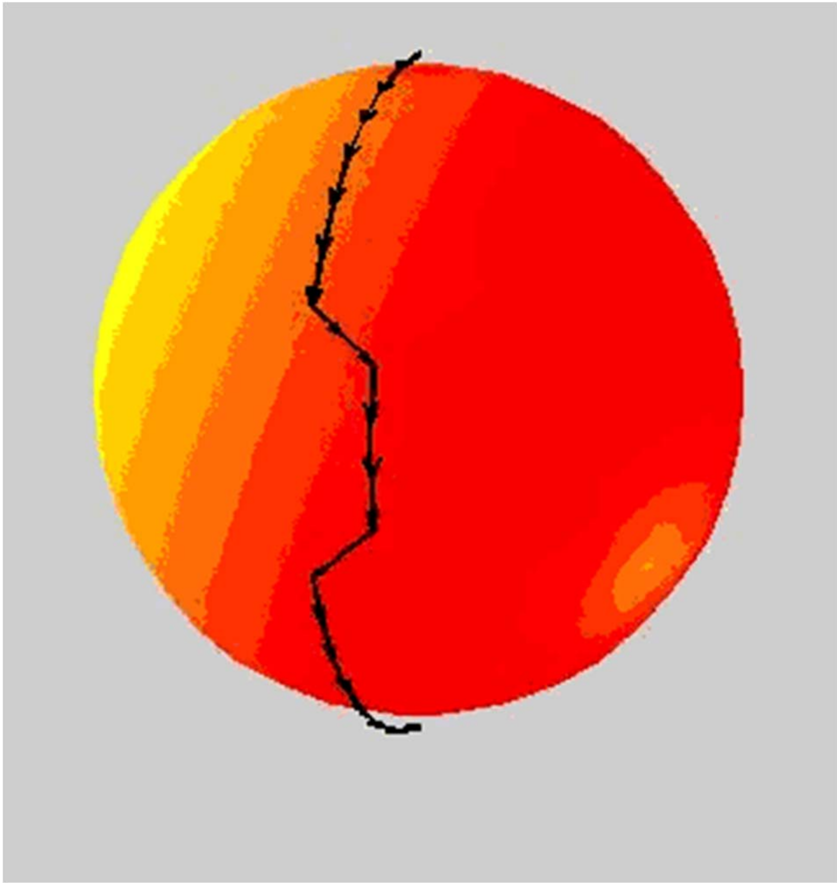
Toroidal field

α **effect**

Poloidal field

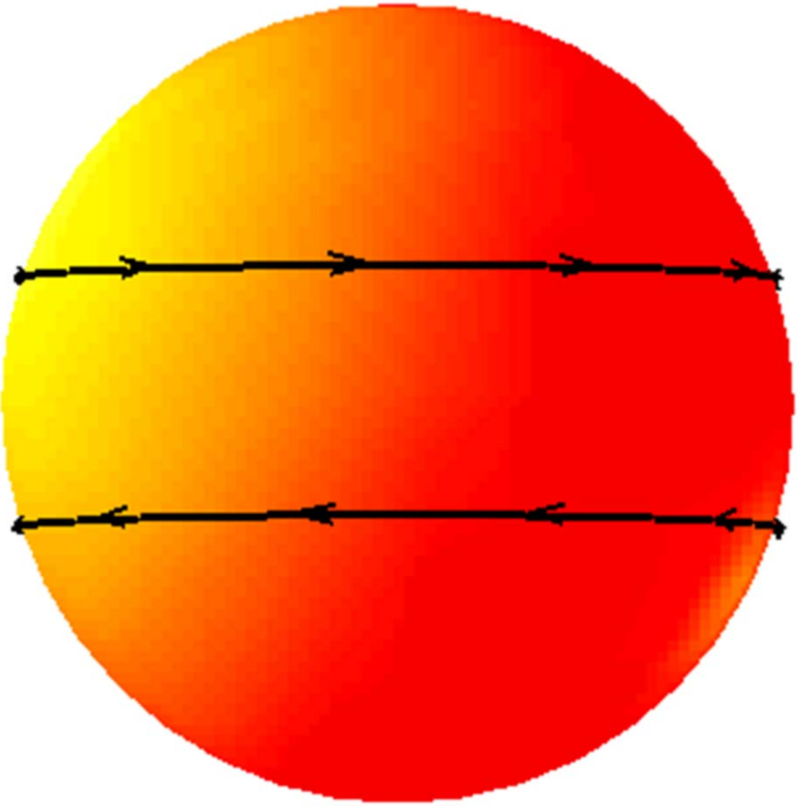
The resulting rising Ω-loop (the name refers to the shape) will be twisted by the Coriolis force produced by the Sun’s rotation. This twisting - which is known as α effect - leads to the top of the loop pointing in the poloidal direction, as shown in the right-hand panel of the above figure. If enough resulting poloidal field elements reconnect, a poloidal field will be reconstructed (with a reversed polarity from the original poloidal field).

Illustration of the Ω-effect



(i) Generation of toroidal field by shearing a pre-existing poloidal field by differential rotation (Ω-effect)

Illustration of the α-effect



(ii) Re-generation of poloidal field by lifting and twisting a toroidal flux tube by helical turbulence (α-effect)

Proposed by Parker (1955)

Mathematically formulated by Steenbeck, Krause & Radler (1969)

Kinematic Dynamo

In the astrophysical context, a dynamo is a fluid flow capable of sustaining a magnetic field indefinitely against Ohmic decay.

Consider a combined action of the α-effect and differential rotation. We shall assume that the kinetic helicity $\alpha(r,\theta)$ and angular velocity $\Omega(r,\theta)$ are known functions.

We consider the mean-field equation

$$\frac{\partial <\mathbf{B}>}{\partial t} = \nabla \times (<\mathbf{v}> \times <\mathbf{B}> + \alpha <\mathbf{B}> - \eta_t \nabla \times <\mathbf{B}>),$$

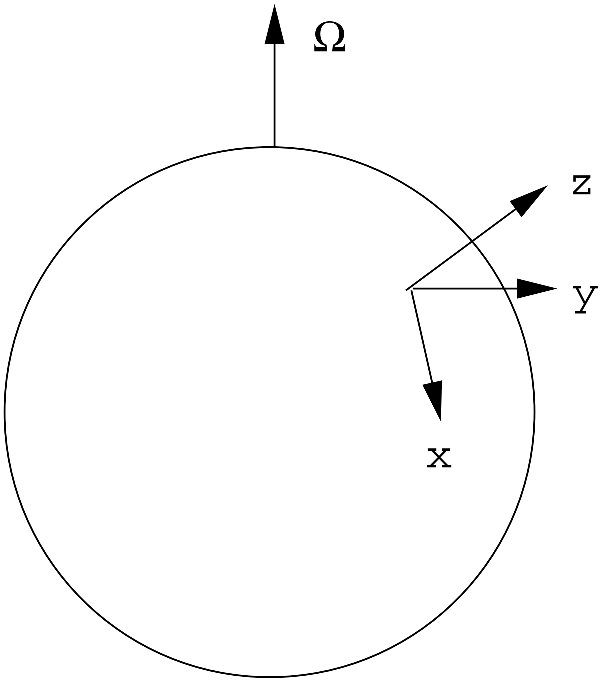
where $\mathbf{v}$ is the rotational velocity, and obtain a simple dynamo solution.

The solution plan:

1. Consider the mean-field induction equation in a local Cartesian coordinate system.
2. Separate the magnetic field vector into two component: poloidal and toroidal
3. Represent the poloidal component in terms of the vector-potential
4. Solve the equations in terms of periodic functions of space and time (similar to the JWKB approximation).

Solution of the mean-field dynamo equation (Parker, 1955)

Consider this equation in cartesian coordinates in a small region of the Sun, so that axis x has direction along meridian, axis y is along the latitude, and axis z is perpendicular to the surface.



Magnetic field can separated in two parts:

- toroidal $\mathbf{B}_t = B_t \mathbf{e}_y$
- poloidal $\mathbf{B}_p = (B_x, 0, B_z)$

The poloidal can be represented in terms of a vector-potential, A, which has only one component, A_y: $\mathbf{B} = \nabla \times \mathbf{A}$,

or $$\mathbf{B}_p = \left(-\frac{\partial A}{\partial z}, 0, \frac{\partial A}{\partial x}\right).$$

The induction equation in terms of A is:

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\partial \nabla \times \mathbf{A}}{\partial t} = \nabla \times [\mathbf{v} \times (\nabla \times \mathbf{A}) + \alpha \mathbf{B} - \eta_T \nabla \times \nabla \times \mathbf{A}].$$

From this equation we obtain equations for A and B_t:

$$\frac{\partial A}{\partial t} = \alpha B_t + \eta_T \nabla^2 A,$$

$$\frac{\partial B_t}{\partial t} = \frac{\partial v}{\partial z}\frac{\partial A}{\partial x} - \frac{\partial v}{\partial x}\frac{\partial A}{\partial z} + \eta_T \nabla^2 B_t.$$

Since $v = r\Omega = (R+z)\Omega$, then $\frac{\partial v}{\partial z} = \Omega.$

Solution of the mean-field dynamo equation (Parker, 1955)

The derivative $\frac{\partial v}{\partial x}$ corresponds to the latitudinal differential rotation. However, in the first approximation we do not consider this term.

Thus, in the simple 1D case we have a system of two equations:

$$\frac{\partial A}{\partial t} = \alpha B_t + \eta_T \frac{\partial^2 A}{\partial x^2}$$

$$\frac{\partial B_t}{\partial t} = \Omega \frac{\partial A}{\partial x} + \eta_T \frac{\partial^2 B_t}{\partial x^2}$$

For constant coefficients α , Ω and η_t we can seek a solution in terms of periodic functions: $A = A_0 e^{-i\omega t + ikx}$, $B_t = B_0 e^{-i\omega t + ikx}$.

Substituting these in the equations we obtain a linear system:

$$(-\omega + \eta_T k^2) A_0 - \alpha B_0 = 0$$

$$(-\omega + \eta_T k^2) B_0 - \Omega ik A_0 = 0.$$

A non-zero solution exists if $(-i\omega + \eta_T k^2)^2 - ik\alpha\Omega = 0.$

Solution of the mean-field dynamo equation (Parker, 1955)

For $\alpha\Omega > 0$:
$$-i\omega + \eta_T k^2 = \pm\sqrt{i}\sqrt{k\alpha\Omega} = \pm\frac{1+i}{\sqrt{2}}\sqrt{k\alpha\Omega}$$

$$-i\omega = \left(-\eta_T k^2 + \sqrt{\frac{k\alpha\Omega}{2}}\right) + i\sqrt{\frac{k\alpha\Omega}{2}}.$$

This is a **dispersion relation for dynamo waves**.
Then the solution for toroidal magnetic field is:

$$B_t = B_0 e^{-i\omega t + ikx} = B_0 \exp\left[\left(\underbrace{-\eta_T k^2 + \sqrt{\frac{k\alpha\Omega}{2}}}_{\text{growth/decay}}\right)t + i\left(\underbrace{\sqrt{\frac{k\alpha\Omega}{2}}t + kx}_{\text{wave term}}\right)\right].$$

It describes dynamo waves traveling poleward (towards negative x).

If we consider the case $\alpha\Omega < 0$, then the propagation is the positive direction, towards the equator.

Magnetic field grows if $\dfrac{\alpha\Omega}{2\eta_T^2 k^3} > 1$, or in terms of **Dynamo Number**, $R_D = \dfrac{\alpha\Omega R^3}{\eta_T^2}$,

the condition for the dynamo instability $R_D (kR)^{-3} > 1$.

Solution of the mean-field dynamo equation (Parker, 1955)

By considering the dynamo equations in the spherical coordinates one can show that the dynamo waves propagate along surfaces Ω=const, and the direction of propagation is given by the vector:

$$\alpha \nabla \Omega \times \mathbf{e}_\phi,$$

where $\mathbf{e}_\phi$ is an azimuthal unit vector.

This is so-called **Parker-Yoshimura rule.**

The magnetic field growth is limited by back reaction on convection: magnetic field changes the properties of convection. This is modelled by **"alpha-quenching"**:

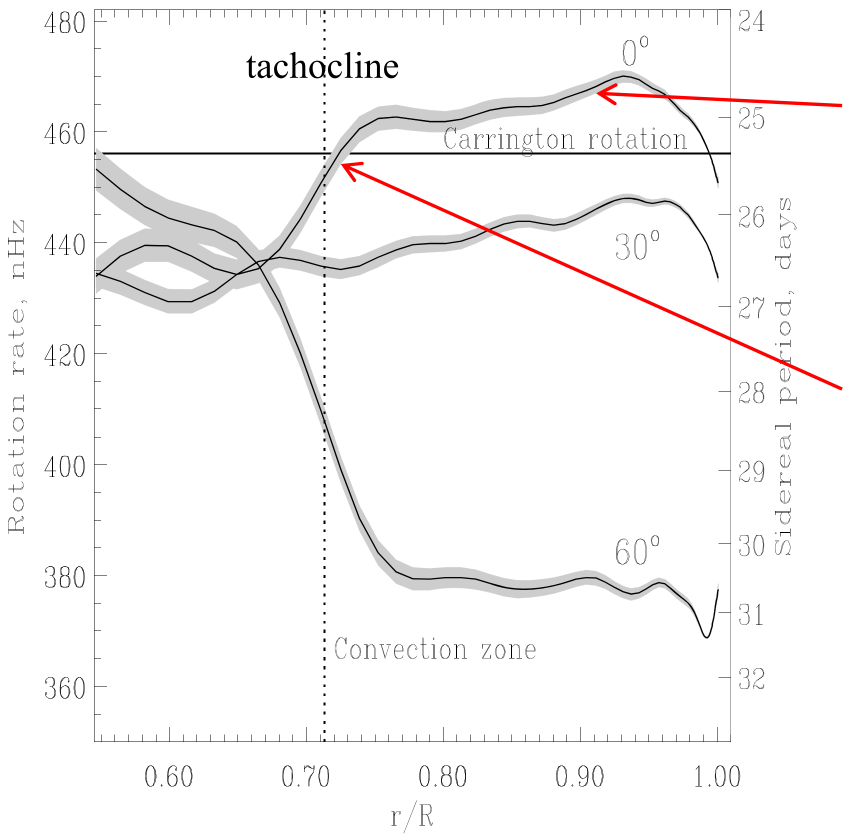
$$\alpha = \frac{\alpha_0}{1 + (B/B_0)^2}.$$

This provides a stationary oscillatory solution.

It was believed that the solar dynamo mostly operates in the tachocline at the low boundary of the convection zone because it is difficult to accumulate strong magnetic field in the convection zone.

Difficulties of the Parker-Yoshimura's model

The equator-ward propagation of the dynamo wave is governed by Parker-Yoshimura sign rule; that is $\alpha \, d\Omega/dr < 0$ in the Northern hemisphere. This can explain the magnetic butterfly diagram. The sign of α is determined by the Coriolis force: $\alpha > 0$ in the Northern hemisphere. Therefore, in 1960's and 70's, it was believed that the solar rotation rate rotation increases inward throughout the convection zone.

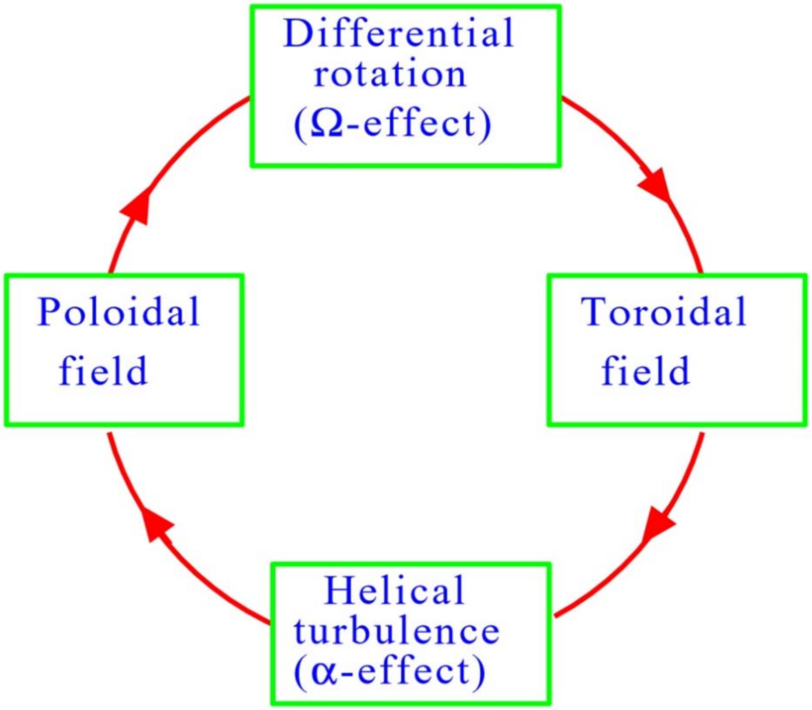


But, in 90's, helioseismic analysis inferred that in the bulk of the convection zone the rotation rate decreases with depth.

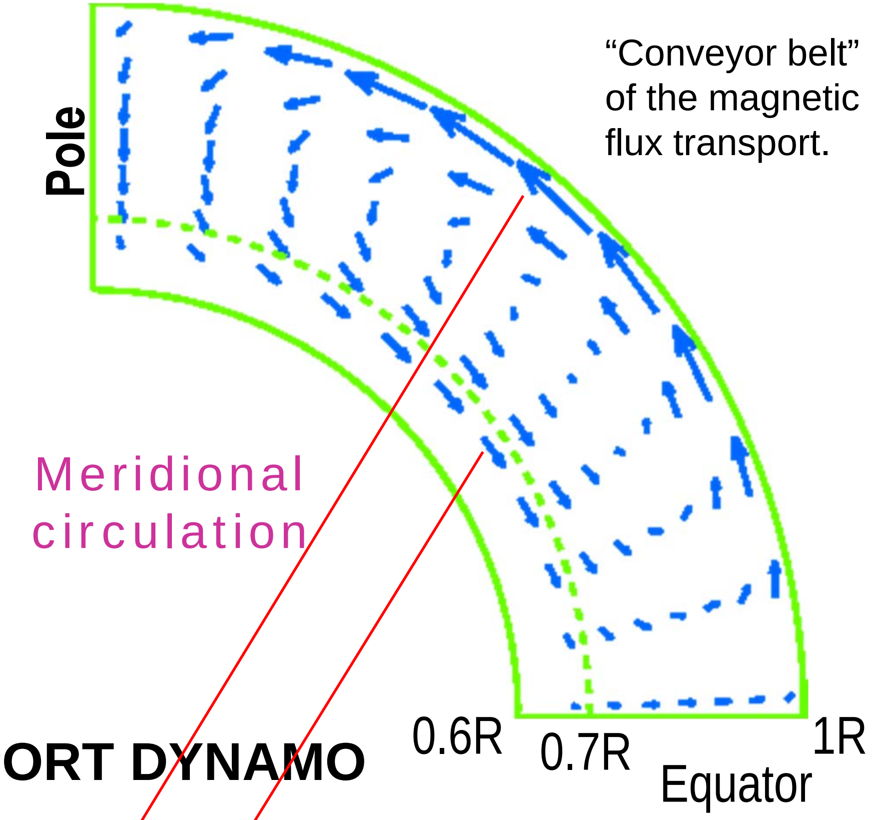
The radial gradient is strong at or below the base of the convection zone in the tachocline, and it is also decreasing inward at the sunspot latitudes (0-30 degrees).

This means that the Parker's dynamo model could not explain the sunspot cycles. Therefore, it was suggested that the dynamo operates in the tachocline, and that the butterfly diagram is due to magnetic flux transport by the meridional circulation – **"flux-transport dynamo model".**

What is a Flux-transport Dynamo?

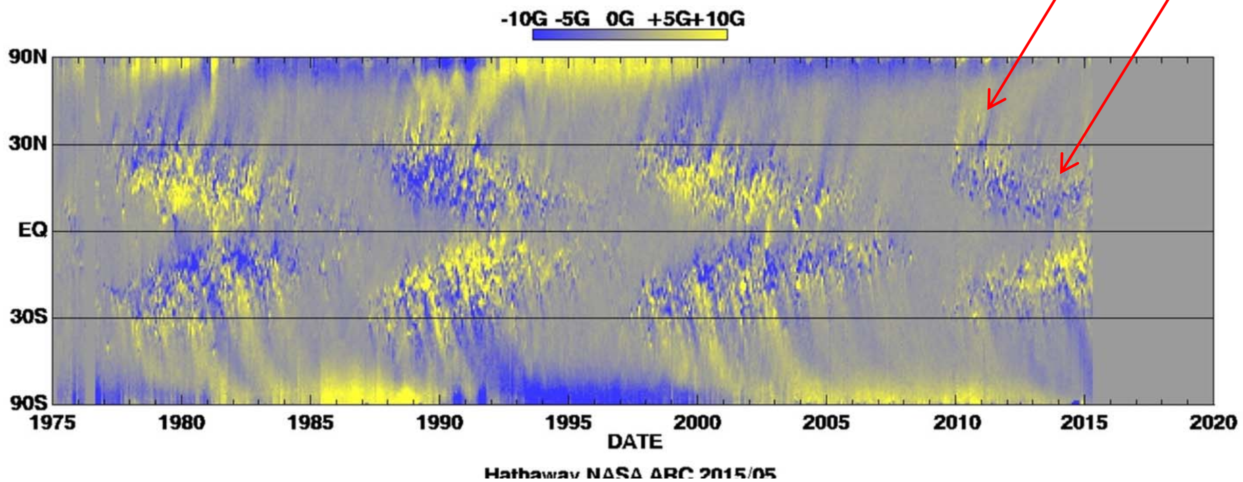


+ Meridional circulation



“Conveyor belt” of the magnetic flux transport.

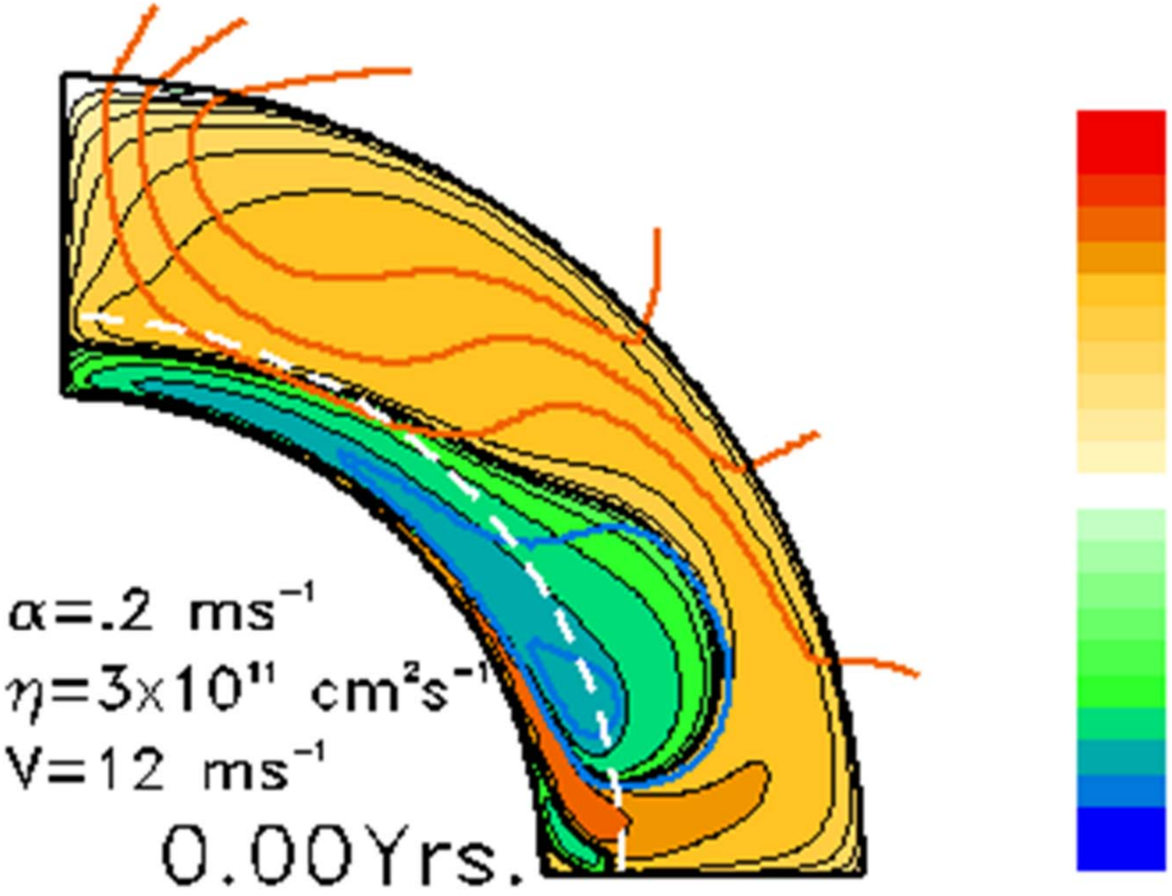
⇒ **FLUX-TRANSPORT DYNAMO**



The butterfly diagram is explained by the flux transport at the bottom of the convection zone.
The polar field reversals are explained by the surface flux transport.

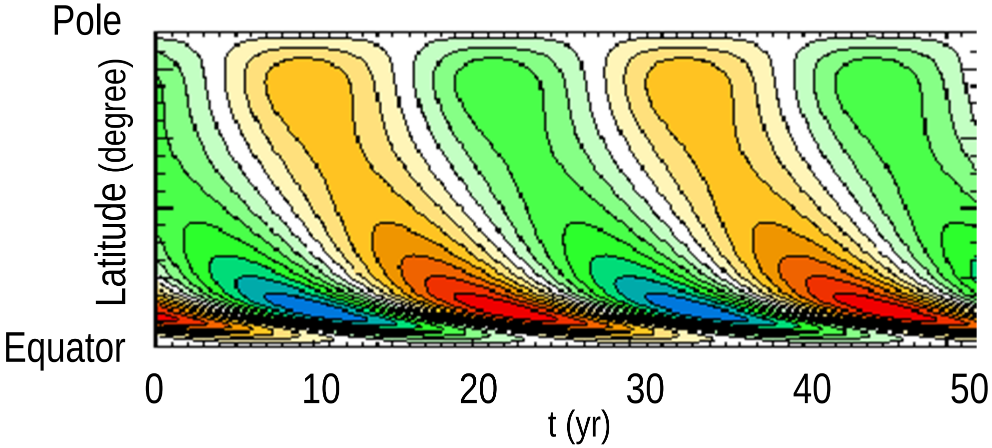
(Courtesy of M.Dikpati)

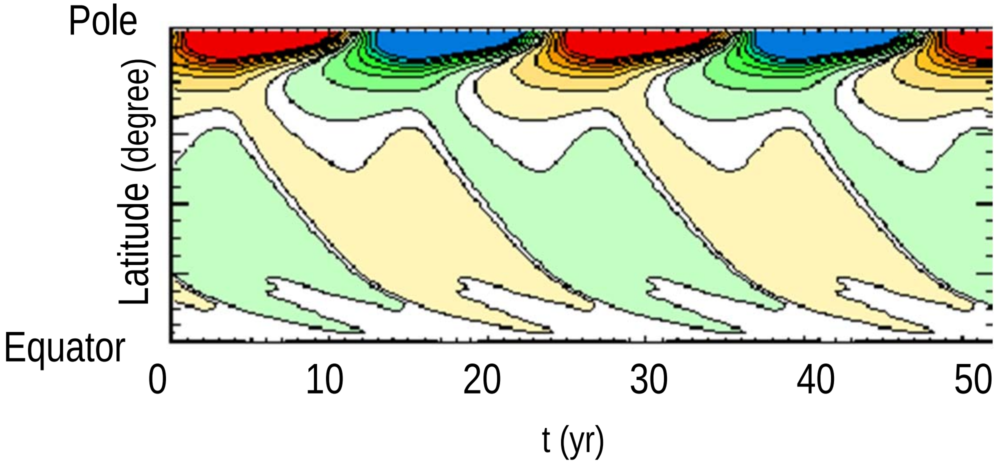
Evolution of Magnetic Fields
In a Babcock-Leighton Flux-Transport Dynamo



(Courtesy of M.Dikpati)

Time-latitude Diagrams Produced from the Babcock-Leighton Flux-transport Dynamo Solution


Toroidal Field at r = 0.7R


Surface Radial Field

Dikpati & Charbonneau, 1999, ApJ, 518, 508

- **Equatorward migrating sunspot belts**
- **Poleward drifting large-scale radial fields**
- **Correct phase relation between these two fields**
- **Dynamo cycle period (T) primarily governed by meridional flow speed**

$$T = 56.8\, v_m^{-0.89}\, s_0^{-0.13}\, \eta_T^{0.22} \,,$$

v_m → max. flow speed

s_0 → surface poloidal source

η_T → turbulent diffusivity

(Courtesy of M.Dikpati)

Conclusions of the flux-transport dynamo model

- **Large-scale solar dynamo mechanism involves 3 basic processes;**

  (i) Ω-effect,
  (ii) α-effect,
  (iii) flux-transport by meridional circulation

- **Mean meridional flow sets the solar clock**

- **Sun is likely to have both Babcock-Leighton type and tachocline α-effect.**

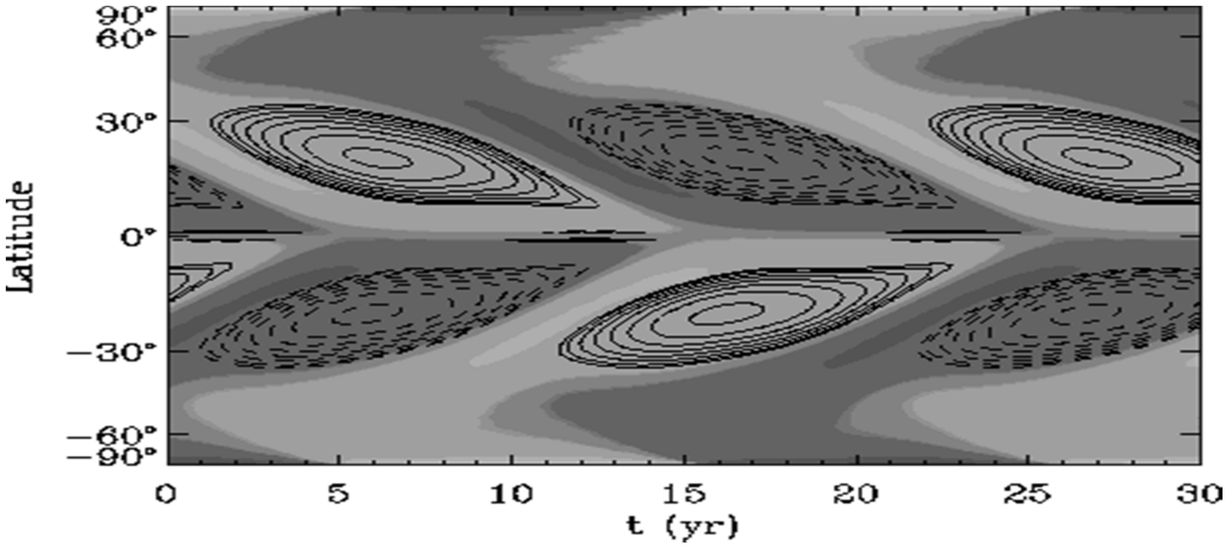
(Courtesy of M.Dikpati)

Validity test of calibration: Time-latitude diagram to match with observation

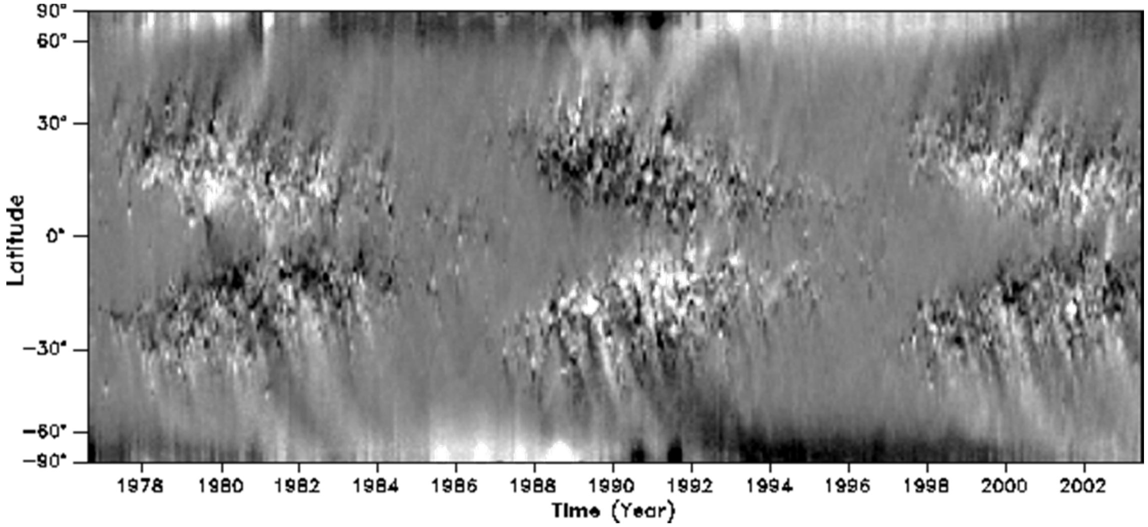
Model output

Contours: toroidal fields at CZ base

Gray-shades: surface radial fields

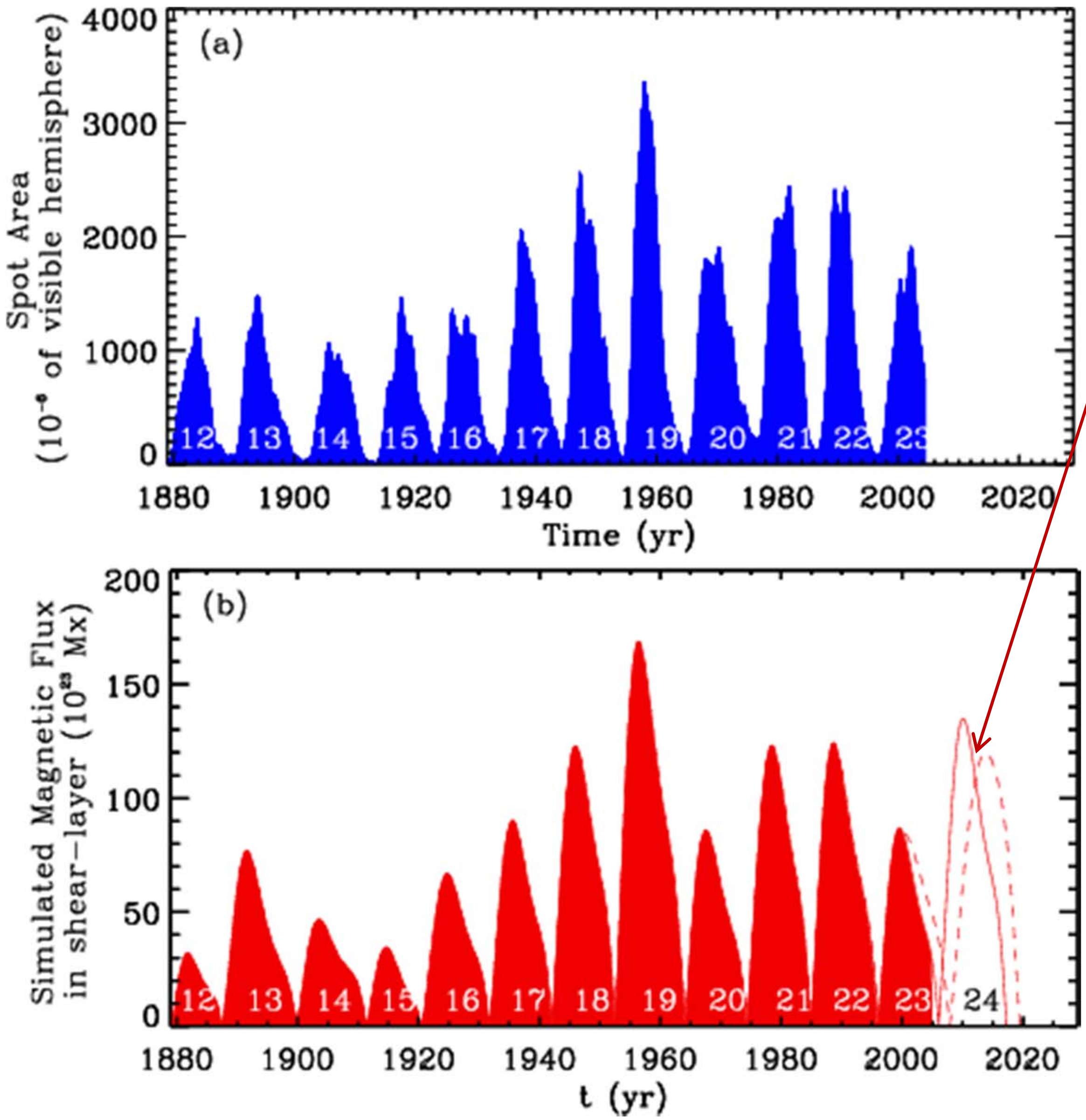


Observed NSO map of longitude-averaged photospheric fields



(Courtesy of M.Dikpati)

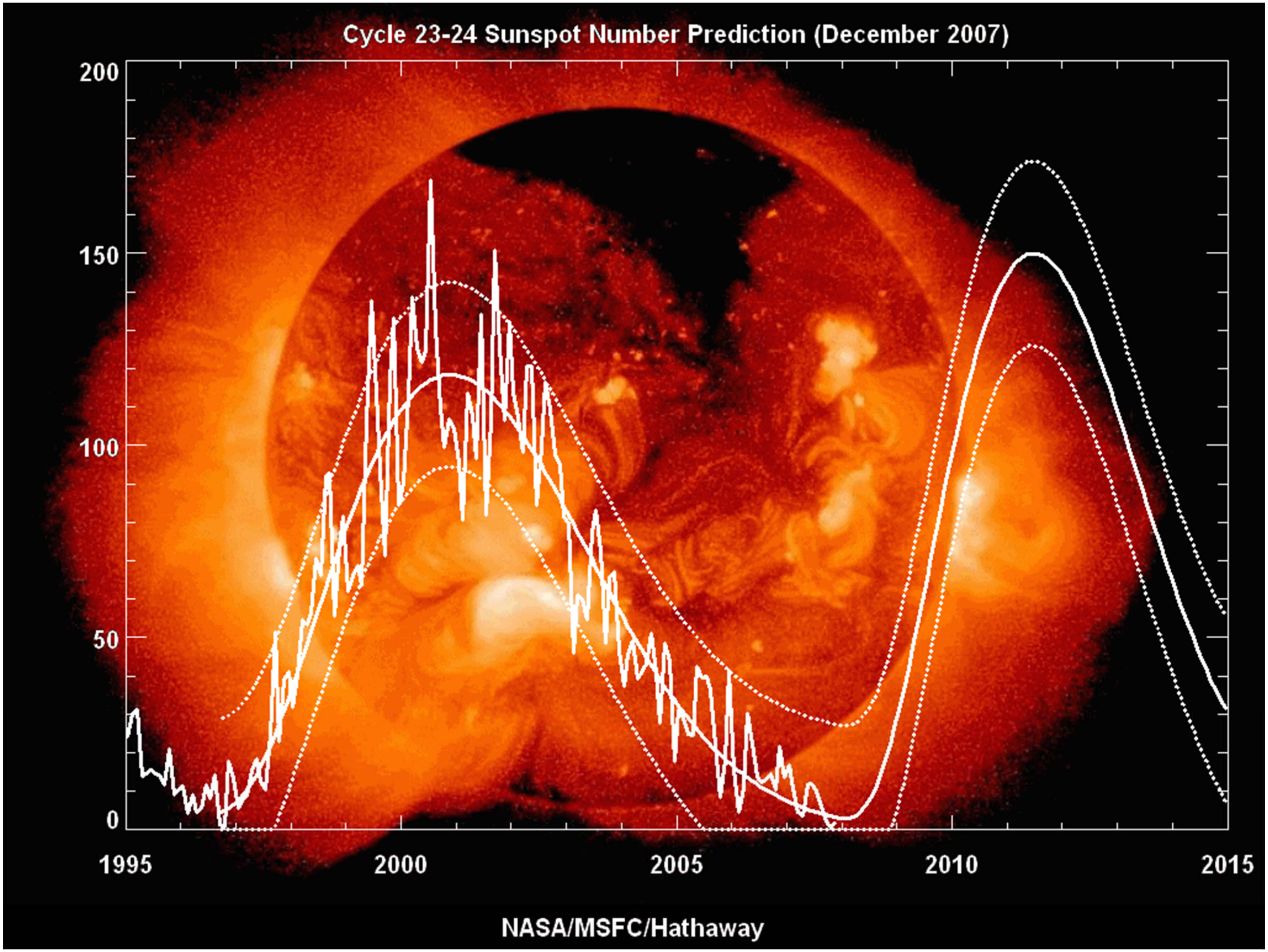
Solar-cycle prediction

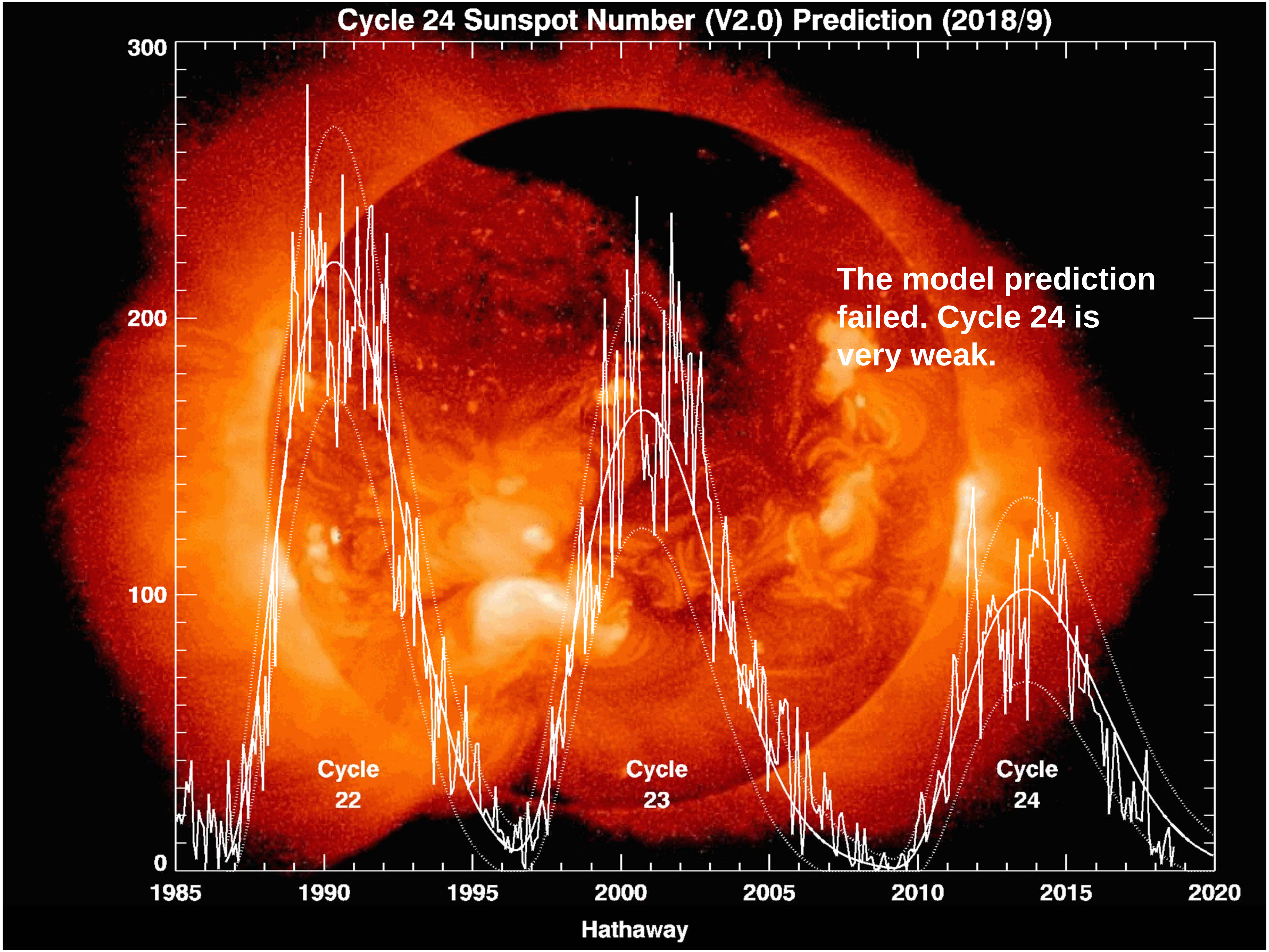


The calibrated flux-transport dynamo model predicted that Solar Cycle 24 will be much stronger than Cycle 23.

Solid curve shows the prediction for constant meridional circulation speed; the dashed curve is the prediction in the case of varying circulation speed.

(Courtesy of M.Dikpati)


Cycle 23-24 Sunspot Number Prediction (December 2007)
200
150
100
50
0
1995
2000
2005
2010
2015
NASA/MSFC/Hathaway



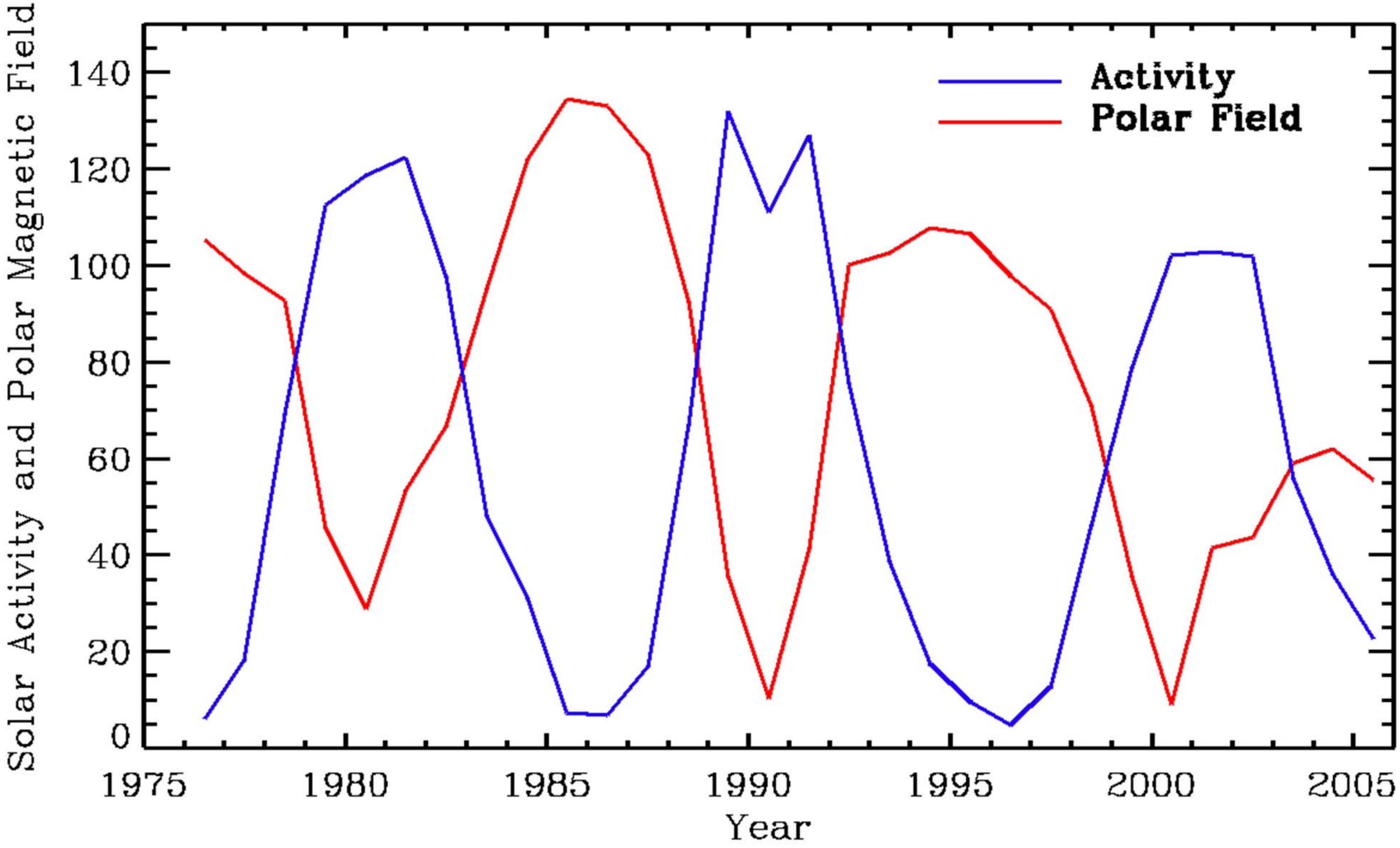
Cycle 24 Sunspot Number (V2.0) Prediction (2018/9)

The model prediction failed. Cycle 24 is very weak.

Hathaway

Solar cycle precursors:

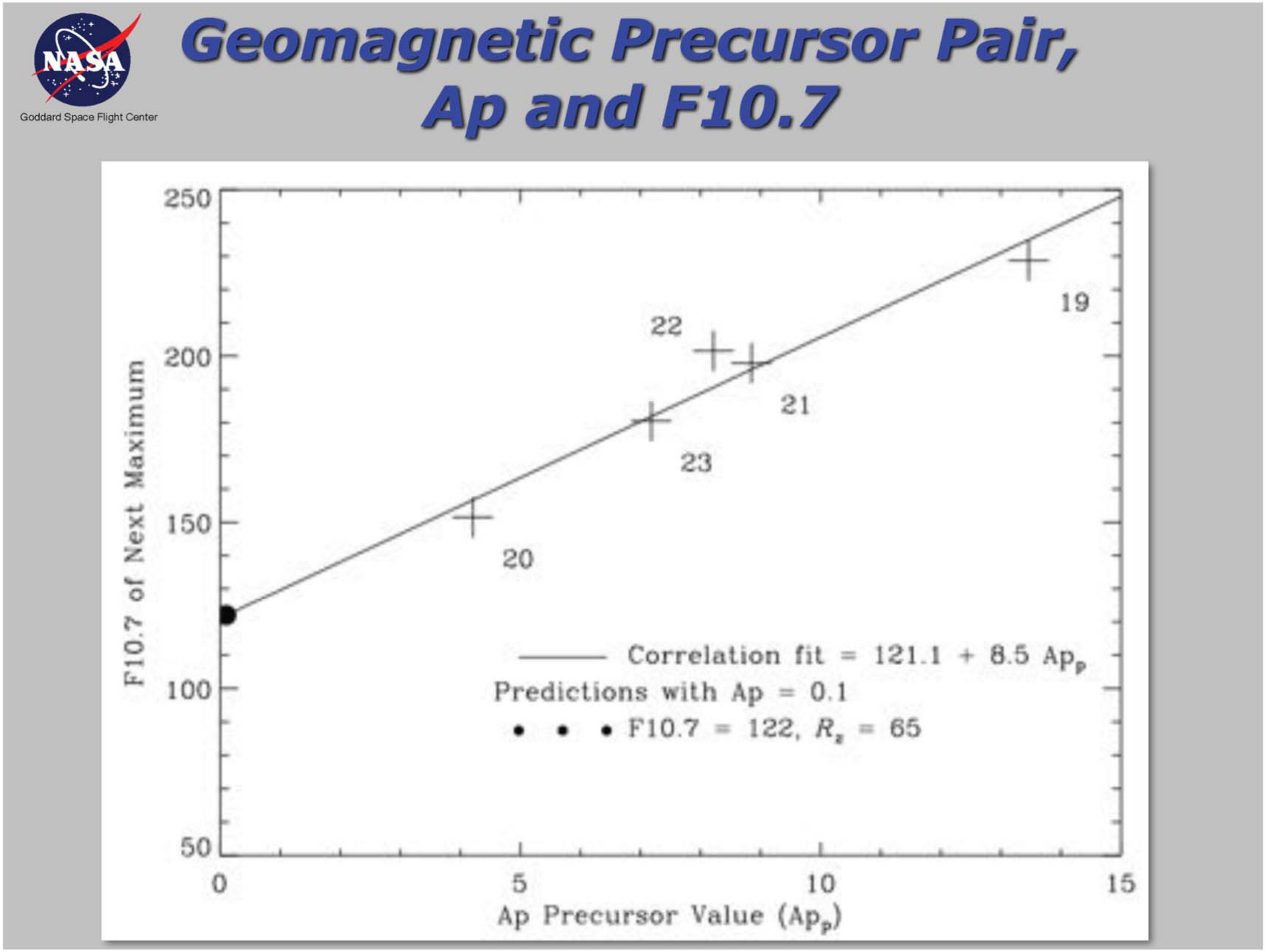
polar magnetic field strength



Solar cycle precursors:

geomagnetic Ap index

Geomagnetic Precursor Pair,

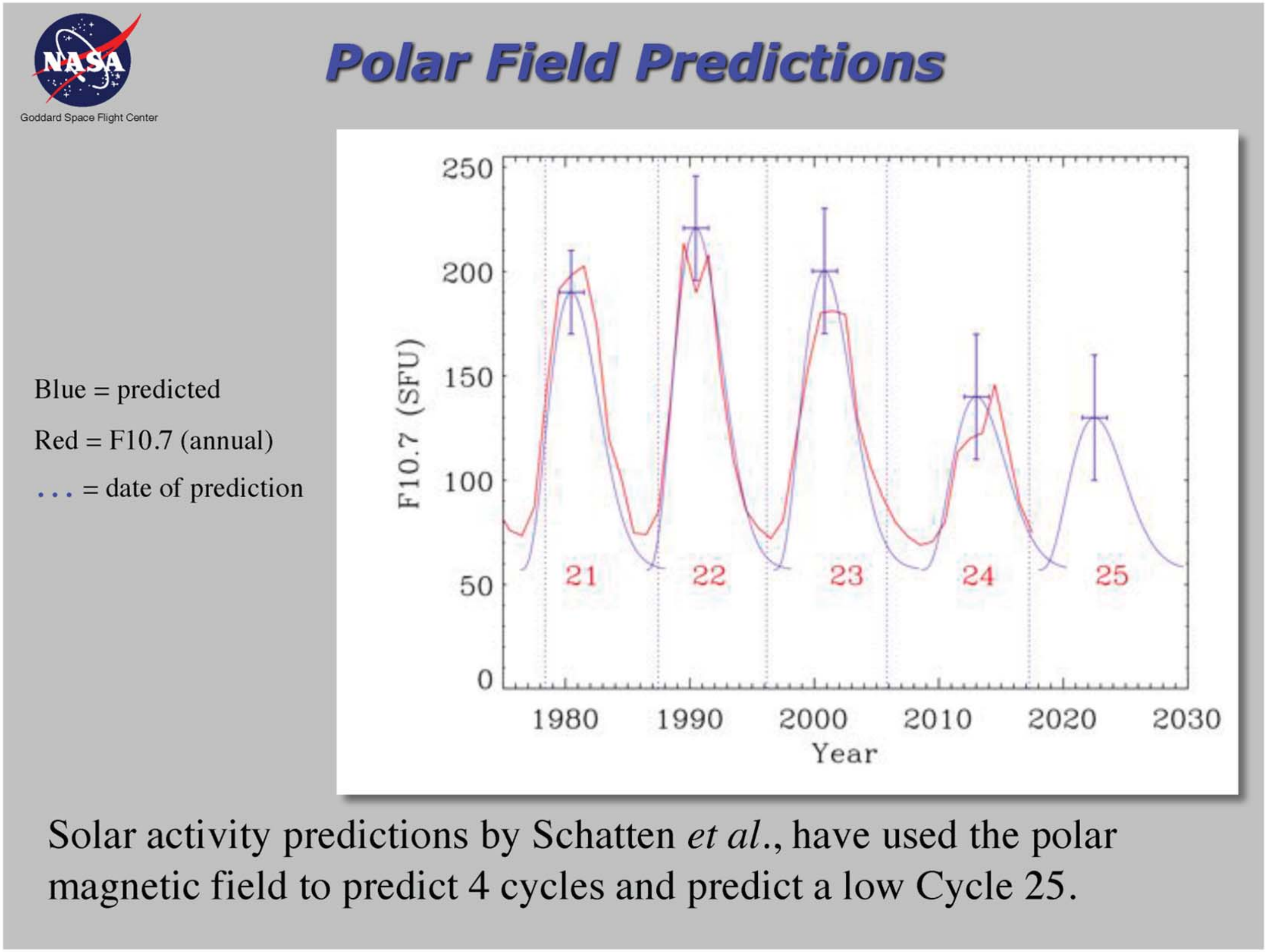


Goddard Space Flight Center

Geomagnetic Precursor Pair, Ap and F10.7

Solar cycle precursors:

polar magnetic field strength



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Polar Field Predictions

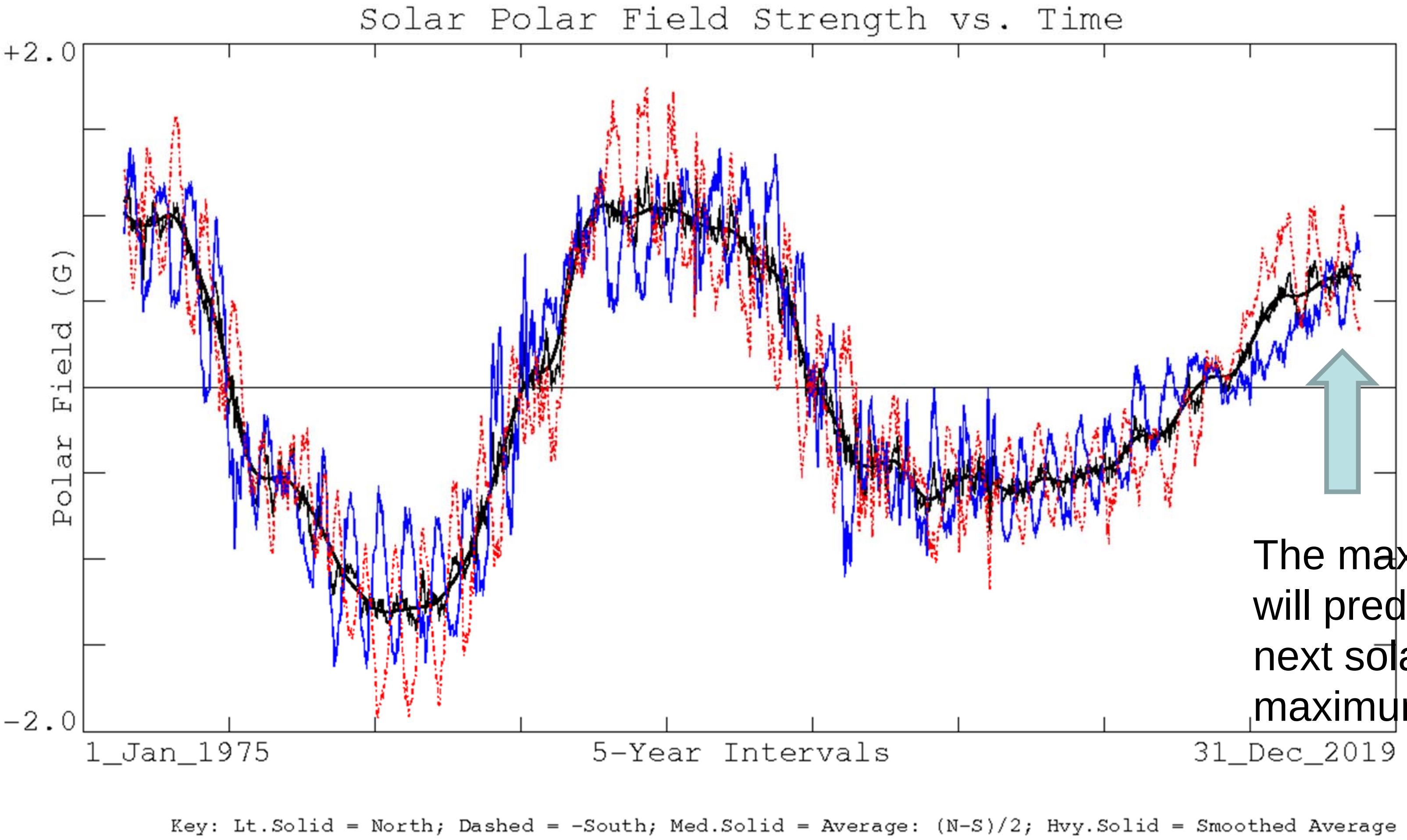
Blue = predicted

Red = F10.7 (annual)

. . . = date of prediction

Solar activity predictions by Schatten *et al*., have used the polar magnetic field to predict 4 cycles and predict a low Cycle 25.

Measurements of the polar magnetic field at Wilcox Solar Observatory



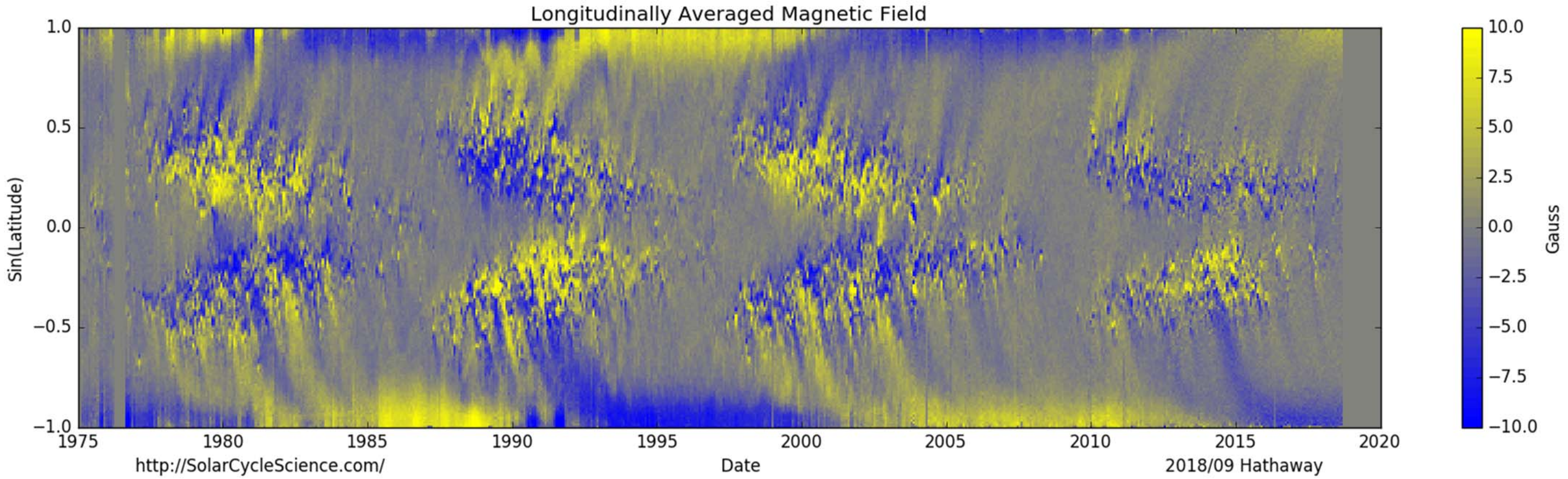
The max value will predict the next solar maximum

Difficulties

1. **A Babcock-Leighton dynamo is not self-excited; won't revive after Maunder minima**

Can there be other poloidal sources to revive the solar dynamo?

2. Furthermore, N & S hemispheres are coupled by an antisymmetric magnetic field about the equator, as inferred from Hale's polarity rule



New ideas in the solar dynamo theories

The discovery of the double-cell meridional circulation is another difficulty of the flux-transport model.

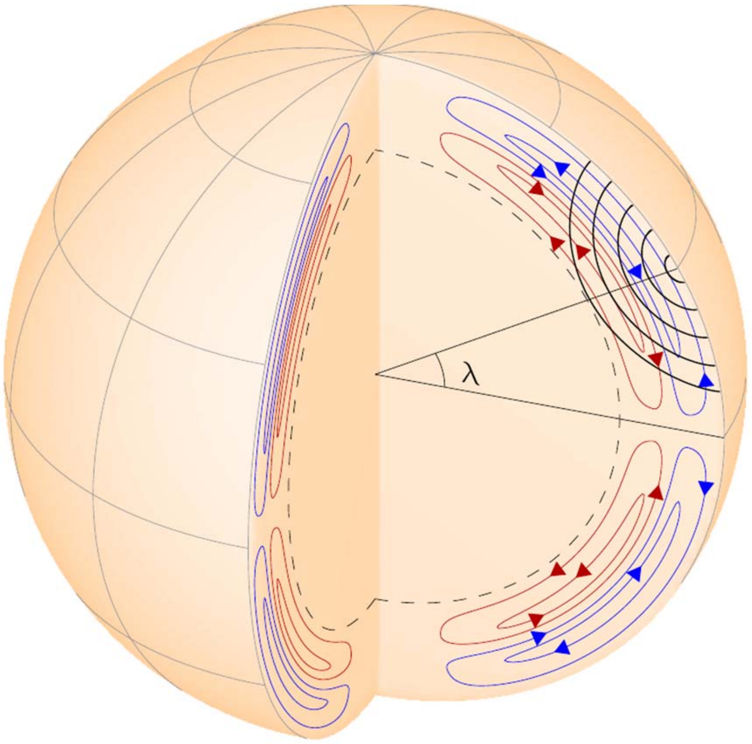
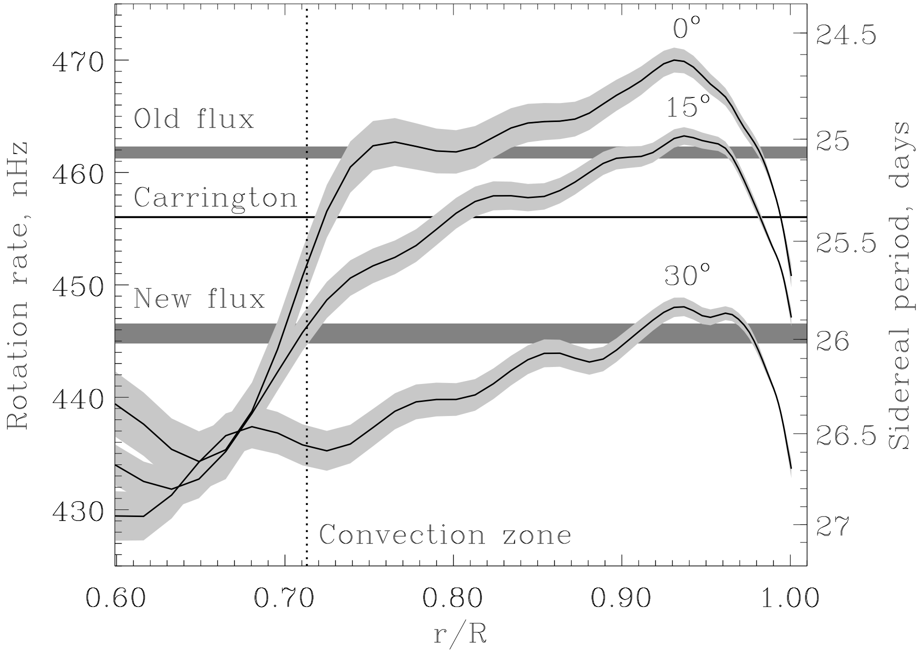


Figure 1. Diagram of the deep-focusing time–distance measurement scheme, with black curves showing some samples of acoustic wave paths. Blue and red streamlines show a schematic structure of the meridional circulation illustrating our results. Black dashed lines show the bottom of the convection zone at 0.7 $R_{\odot}$.

Zhao et al, 2013

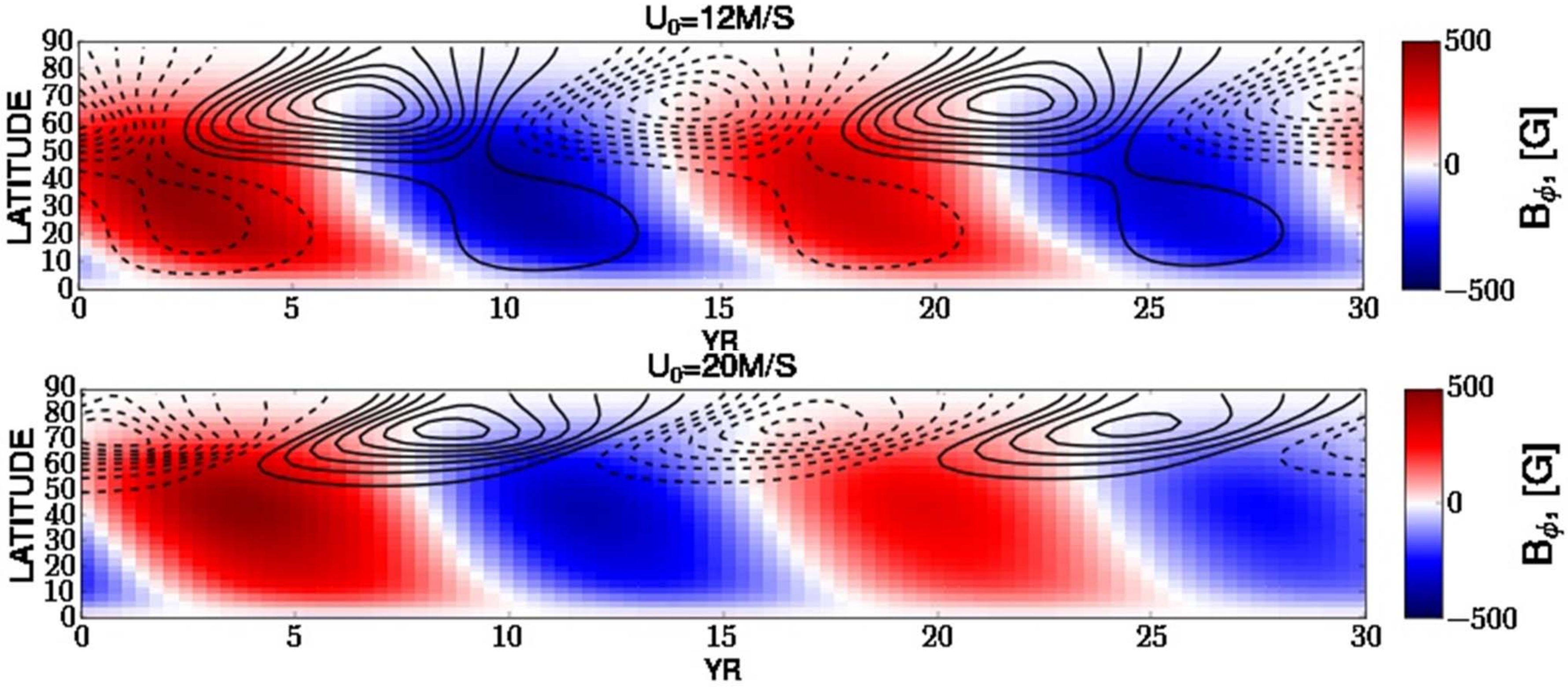
New idea: the solar dynamo works in the whole convection zone, but the butterfly diagram is formed by the Parker-Yoshimura mechanism in the near-surface shear layer (Brandenburg, 2005).



Magnetic butterfly diagram for the distributed dynamo models with the double-cell meridional circulation

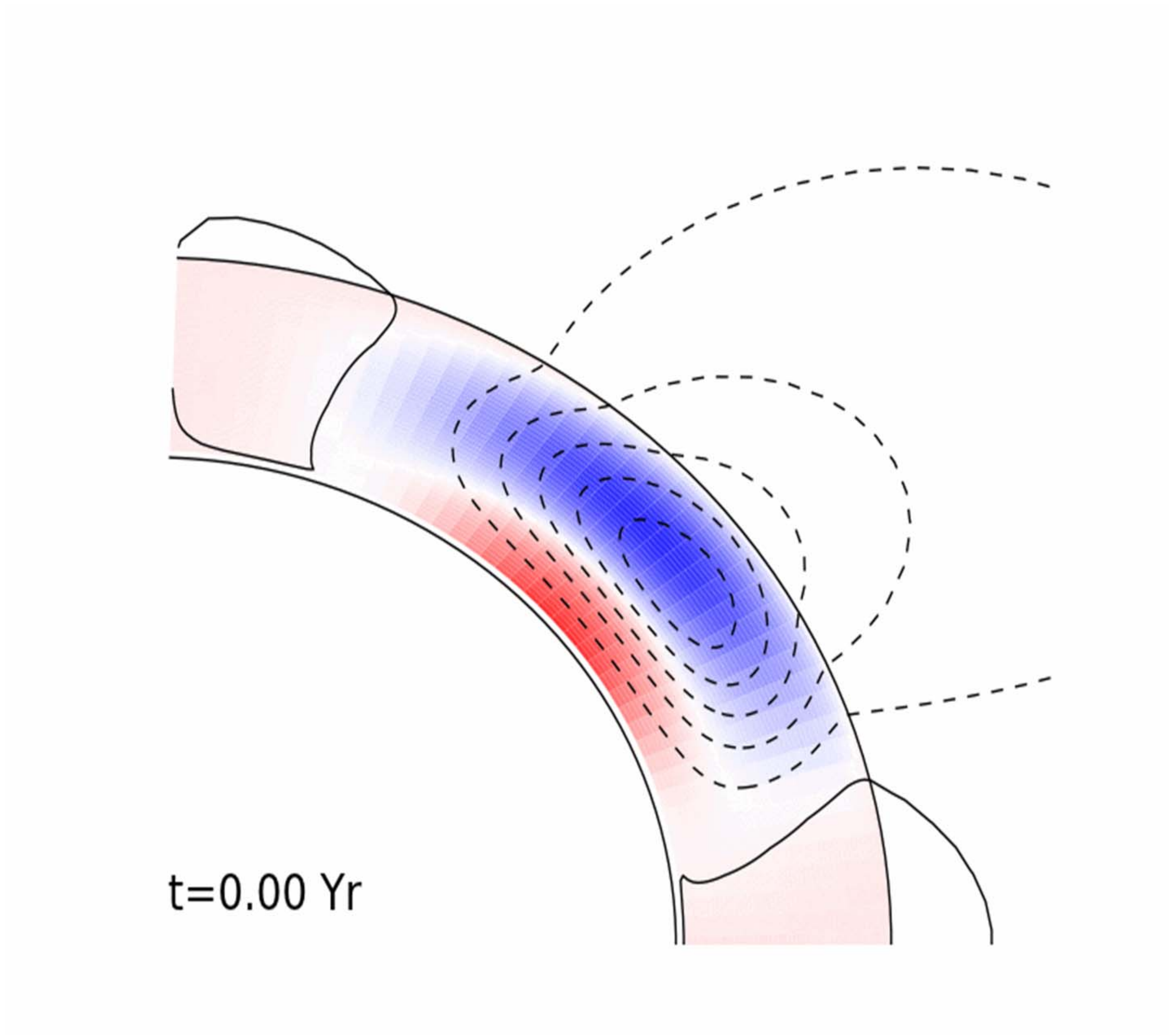
(colors - toroidal magnetic field, contours – radial field)

r=0.82R



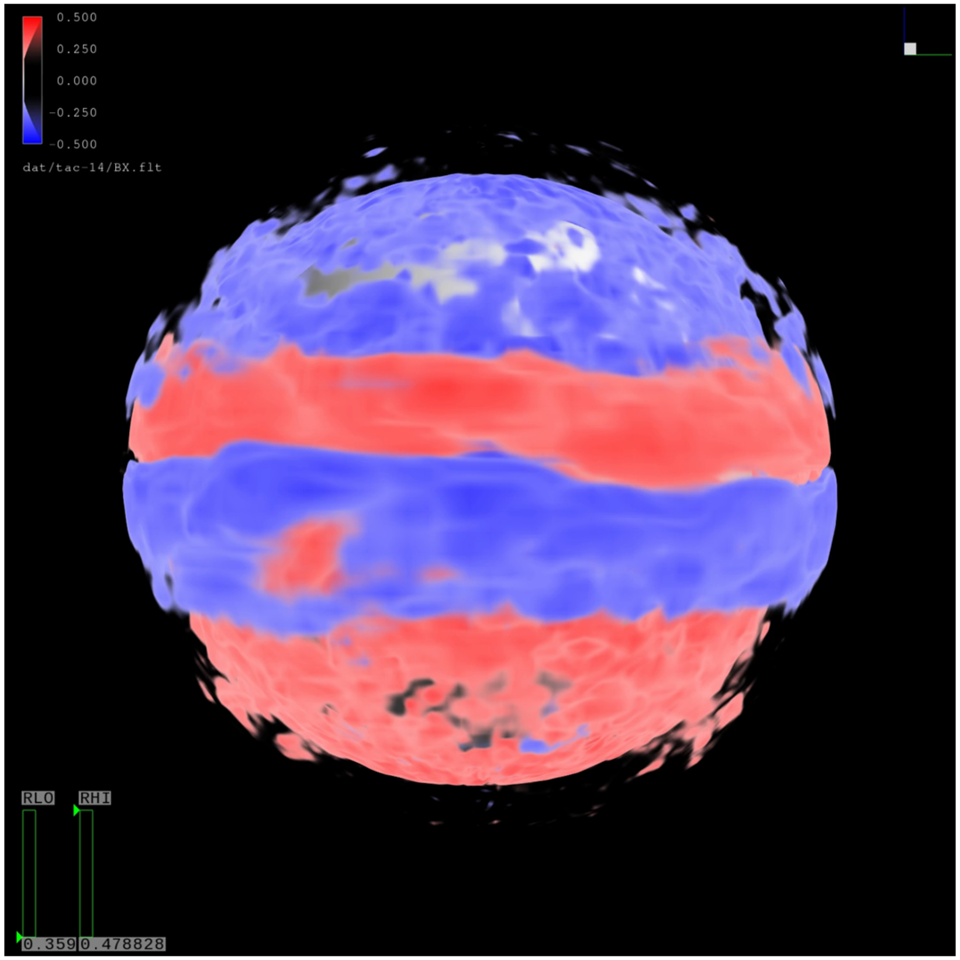
The butterfly diagram can be explained by the dynamo wave in the near-surface rotational shear layer (Pipin & Kosovichev, 2013)

Evolution of magnetic field in the solar interior in the dynamo model with the field migration in the near-surface rotational shear layer



(V. Pipin 2013)

Future development of the dynamo theory: understanding of the turbulence physics, differential rotation and the alpha-effect from 3D numerical MHD simulations



Evolution of the toroidal magnetic field in the convection zone (A.Stejko)